



## Generation of Tunable Focal Spot and Focal hole by Radially Polarized Axisymmetric Bessel-modulated Gaussian beam

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### Abstract

In recent years, an Axisymmetric Bessel-modulated Gaussian beam with quadratic radial dependence (QBG beam) has attracted very much. In this paper, Focal shift in radially polarized QBG beam with radial variance phase wavefront is investigated theoretically by vector diffraction theory. The wavefront phase distribution is cosine function of radial coordinate. Simulation results show that the intensity distribution in focal region of the radially polarized QBG beam can be adjusted considerably by small beam parameter  $\alpha$  and phase parameter  $C$ . On increasing  $C$ , focus can shift along optical axis and focal pattern changes remarkably. When  $\alpha$  increases, the focal spot may change to focal hole that also shift remarkably on increasing  $C$ . Focal shift distance fluctuates on increasing  $C$ , and fluctuation amplitude also changes simultaneously.

### 1. INTRODUCTION

Polarization is one important property of light. This vector nature of light and its interactions with matter make many optical devices and optical system designs possible. The radially polarized beam has gained much interest recently due to its novel properties and applications such as particle-trapping, optical data storage, laser machining, and lithography [1–8] as well as in biological studies [9]. The polarization propagation and light intensity distribution in focal region plays an important role in many optical systems. It was demonstrated both theoretically and experimentally [5]. Optical intensity distributions in focal region have found applications in optical micro-manipulation domain [10 - 14]. Effect of a pure phase-shifting

shifting apodizer in focal region is investigated [15]. Bessel-modulated Gaussian beams with quadratic radial dependence (QBG beam) are a novel class of beam expressed in cylindrical coordinate system. Since QBG beam was introduced recently by Caron and Potvliege [16], it has attracted much attention [16 - 22]. It was shown that such class of beams has familiar collinear geometry of the Gaussian beam and also an interesting non-Gaussian feature for certain values of its parameters [21, 22]. It should be noted that the zeroth-order QBG beam, which is usually referred to as the axisymmetric QBG beam, can be expanded in Laguerre–Gauss modes and has a very flat axial profile when the beam parameter is of order of unity [16, 20, 21, 23], Since then more attention has been paid to the focusing of the beams [24, 25, 26]. Young worth and Brown

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calculated cylindrical-vector fields, near the focal region of an aplanatic lens, and briefly discussed some applications. It is shown that, in the particular case of a tightly focused radially polarized beam, the polarization shows large inhomogeneities in the focal region, while the azimuthally polarized beam is purely transverse even at very high numerical apertures [27].

Zhan and Leger reported a focus shaping technique using generalized cylindrical vector beam. Radially polarized beam is also used to implement resolution enhancement in optical focusing systems [2]. In our knowledge, almost all QBG beams in previous papers are in scalar form without considering the polarization distribution of optical field. In fact, the polarization is very important characteristics to alter propagating and focusing properties of beams, and have attracted many researchers recently [1, 2, 28 – 30]. In addition, in high numerical aperture optical systems, polarization characteristics of beam should be considered to provide more accurate results, and for this case, vector diffraction theory is employed to replace scalar diffraction theory.

In order to get insight into properties of radially polarized QBG beam, the present paper is aimed at studying focal shift of the radially polarized axisymmetric QBG beam with radial variance phase wavefront. The principle of the focusing radially polarized axisymmetric QBG beam is given in Section 2. Section 3 shows the simulation results and discussions. The conclusions are summarized in Section 4.

## 2. THEORY

In the focusing system we investigated, focusing beam is radially polarized axisymmetric QBG whose value of transverse optical field is same as that of the scalar axisymmetric QBG [1 – 3], and its polarization distribution on radial [3]. Therefore the cylindrical coordinate system  $(r, \tilde{O}, 0)$

cylindrical coordinate system  $(r, \tilde{O}, 0)$  the field distribution  $E(r, \tilde{O}, 0)$  of the radially polarized axisymmetric QBG beam at the plane  $z = 0$  is written as,

$$E_0(r, \varphi, 0) = E_0(r, \varphi, z=0) \cdot \left[ \cos(\phi(r)) \cdot n_r \right] \quad (1)$$

Where  $\tilde{O}(r)$  is the polarization angle from radial direction, and is the function of radial coordinate for the radially polarized axisymmetric QBG beam, where  $n_r$  is radial unit vector.

$$E_0(r, \varphi, z=0) = J_0\left(\frac{\mu r^2}{\omega_0^2}\right) \cdot \exp\left(\frac{r^2}{\omega_0^2}\right) \quad (2)$$

Where  $J_0$  denotes the Bessel function of order zero,  $\omega_0$  is the waist width of the Gaussian beam,  $\mu$  is a beam parameter which is complex valued in general. After some simple derivation [30], Eq. 2 can be rewritten as

$$E_0(r, \varphi, z=0) = J_0\left(\frac{\mu \sin^2(\theta)}{w^2 \cdot NA^2}\right) \cdot \exp\left(\frac{\sin^2(\theta)}{w^2 \cdot NA^2}\right) \quad (3)$$

Parameter  $w = \omega_0/r_0$  is called relative waist width, where  $r_0$  is radius of incident optical aperture.  $NA$  is numerical aperture of the focusing system.  $\theta$  is polar angle corresponding to radial coordinate.  $\tilde{O}(r)$  is the polarization characteristics of the focusing radially polarized axisymmetric QBG beam and can be expressed as

$$\phi(\theta) = C \cdot \frac{\tan(\theta) r}{\tan(\alpha)} \cdot \pi \quad (4)$$

Where  $\alpha = \arcsin(NA/n)$  is the convergence angle of the focusing optical system. If the focusing optical system investigated is in air, i.e. refractive index  $n = 1$ .  $C$  is radial variance parameter indicating polarization radial degree of the wave front phase distribution. Using the same analysis method as that in references [1 – 5, 31], the electric field in focal region of radially polarized axisymmetric QBG beam is,

$$E(r, \varphi, z) = E_r e_r + E_z e_z \tag{5}$$

$$E_r(r, z) = A \int_0^\alpha \sqrt{\cos(\theta)} \sin(2\theta) E(\theta) J_1(kr \sin \theta) \exp(ikz \cos \theta) d\theta \tag{6.a}$$

$$E_z(r, z) = 2iA \int_0^\alpha \sqrt{\cos(\theta)} \sin^2(\theta) E(\theta) J_0(kr \sin \theta) \exp(ikz \cos \theta) d\theta \tag{6.b}$$

Where A is a constant,  $J_0(x)$  and  $J_1(x)$  denote zero order and first order Bessel functions of the kind. K is the wave number and  $k = 2\pi/\lambda$ . With  $\lambda$  being the wavelength of illuminating beam.

Substitute Eq. (3) and (4) into Eqs. (6) then substitute Eqs. (6) into Eq. (5), we can obtain the analytical total field in focal region. The optical intensity in focal region is proportional to

the modulus square of Eq. (5), so the focusing properties of radially polarized axisymmetric QBGbeam can be investigated theoretically.

### 3. RESULTS AND DISCUSSIONS

Without loss of validity and generality, it was proposed that  $NA = 0.95$  and  $A = 1$ . The focusing properties of radially polarized

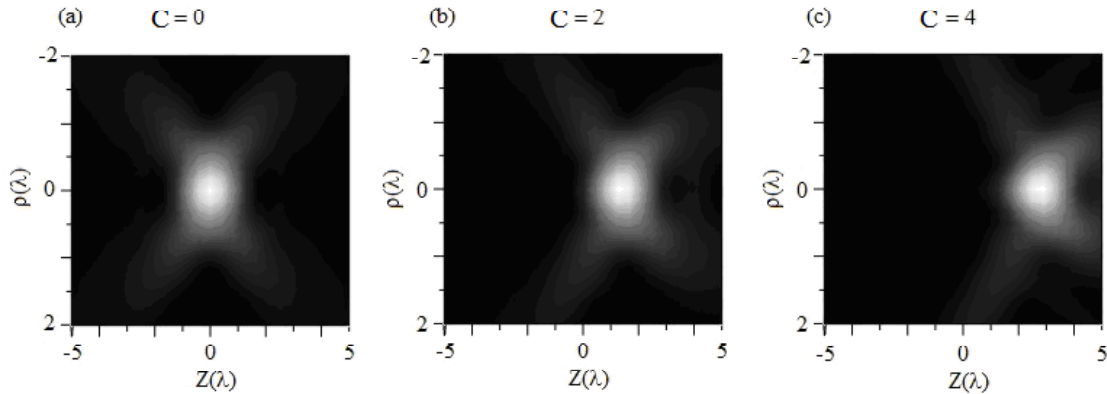


Fig. 1. Intensity distributions in focal region for  $\mu = 0$  and (a)  $C = 0$ , (b)  $C = 2$ , and (c)  $C = 4$  respectively.

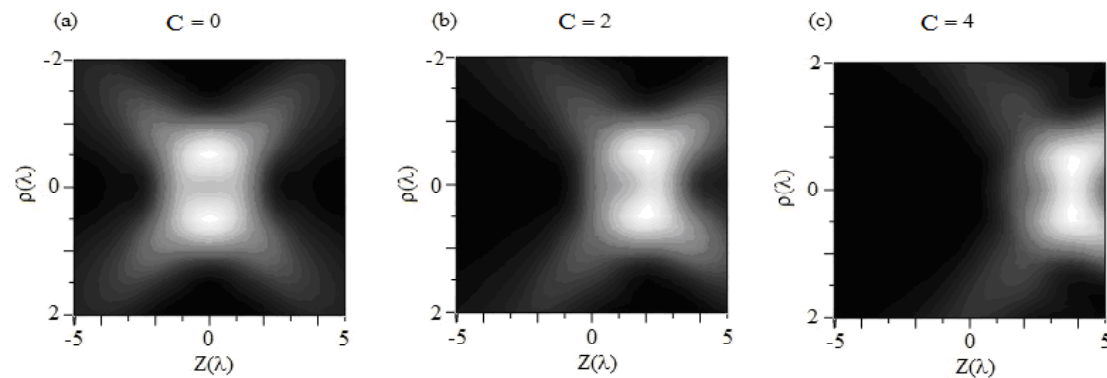


Fig. 2. Intensity distributions in focal region for  $\mu = 3$  and (a)  $C = 0$ , (b)  $C = 2$ , and (c)  $C = 4$  respectively

axisymmetric QBG beam is investigated theoretically, for different beam parameter and different radial variance phase wavefront. Firstly, the intensity distributions in focal region of the radially polarized QBG beam are calculated under condition of  $\mu=0$  and different  $C$ , and are illustrated in Fig. 1. Here it should be noted that the unit of coordinates in all figures in this article is  $k^{-1}$ , where  $k$  is wave number. Focus refers to the maximum optical intensity peak, and focal shift denotes the movement of this focus.

one focal spot in focal region for  $C = 0$ . And the focus shifts along optical axis on increasing radial variance wavefront parameter  $C$ , namely focal shift phenomenon occurs. Focal spot shifts towards optical aperture along optical axis, as shown in Fig. 1(b) for  $C = 2$ . On increasing  $C$  continuously, the focal shift phenomenon occurs without changing direction and focal pattern, illustrated in Fig. 1(c). Now the value of radial wavefront parameter  $C$  is changed to investigate its effect on focal pattern evolution. The intensity distributions for beam parameter  $\mu = 3$  are calculated and shown in Fig. 2.

It can be seen from Fig. 1(a) that there is

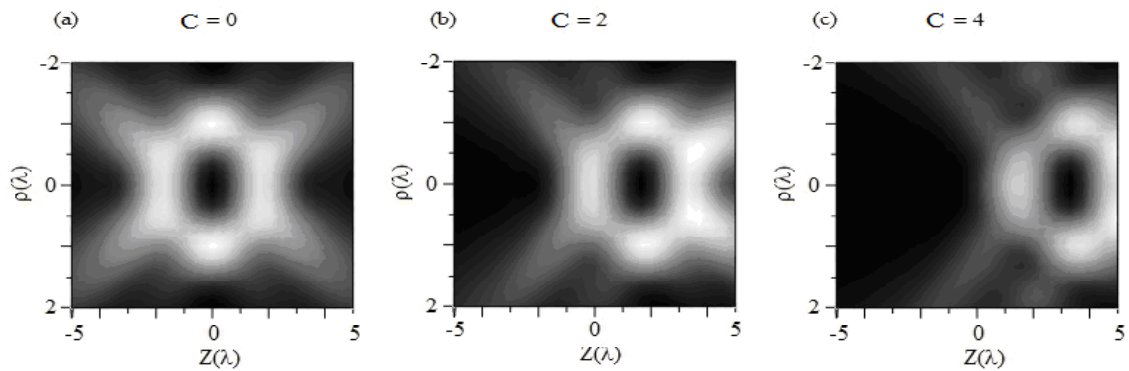


Fig. 3. Intensity distributions in focal region for  $\lambda = 6$  and (a)  $C = 0$ , (b)  $C = 2$ , and (c)  $C = 4$  respectively.

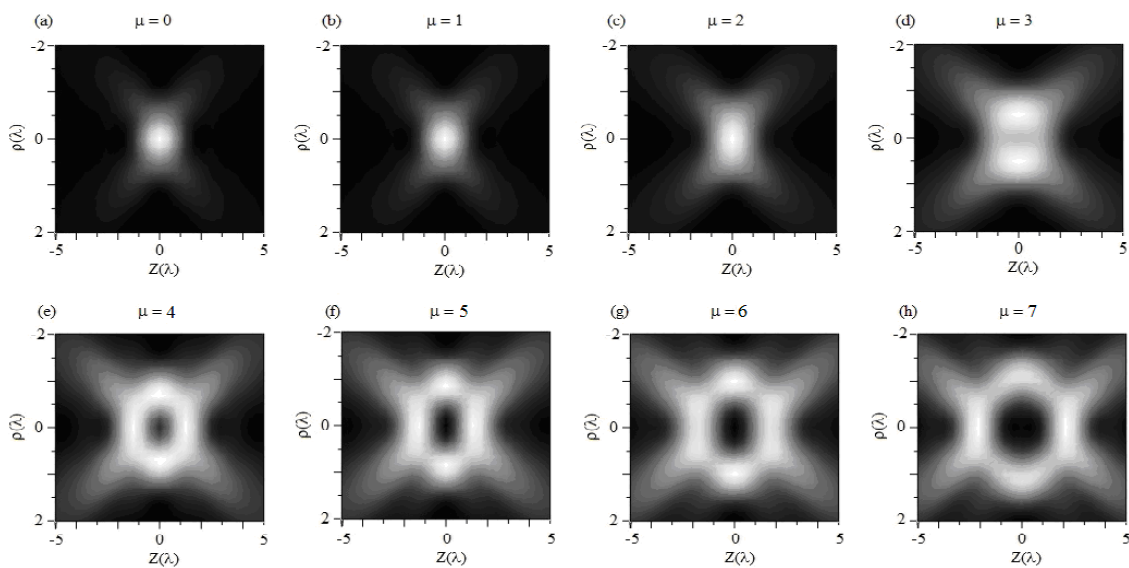


Fig. 4. Intensity distribution in the focal region for different  $\mu$  a)  $\mu = 0$ , b)  $\mu = 1$ , c)  $\mu = 2$ , d)  $\mu = 3$ , e)  $\mu = 4$ , f)  $\mu = 5$ , g)  $\mu = 6$  and h)  $\mu = 7$ .

It can be seen from Fig. 2(a) that there is two focal spot in focal region for  $C = 0$ . And the focus shifts along optical axis on increasing  $C$  from 0 to 2 shown in fig 2(b). On increasing  $C$  continuously, the focal shift phenomenon occurs without changing direction and focal pattern changes evolutionally, illustrated in Fig. 2(c).

The intensity distributions for beam parameter  $\mu = 6$  are calculated and shown in Fig. 3.

When  $\mu = 6$  the system generates one dark hallow focus in the focal region it can be seen from Fig. 3(a) for  $C = 0$ . Upon increasing  $C$  the dark hallow focus starts shifting along optical axis shown in fig 2(b). On increasing  $C$  continuously, the focal shift phenomenon occurs without changing direction and focal pattern, illustrated in Fig. 3(c).

Hence in order to study the effect of beam parameter  $\mu$ , the intensity distributions in focal region of radially polarized QBG with radial variance phase wavefront are investigated under condition of different beam parameter  $\mu$  and it is illustrated in Fig.4

From Fig. 4, it can be seen that there is only one intensity peak for small beam parameter  $\mu = 0$  shown in Fig 4(a). When beam parameter  $\mu$  increases from 0 to 1, focal pattern evolves very considerably, one intensity peak changes into one intensity sphere shell because the intensity in the former focus position shrinks sharply, as shown in Fig. 4(b & c). When  $\mu$  increases from 2 to 3, system generates two intensity peaks with less intensity in center. Upon increasing  $\mu$  from 3 to 4, the focal pattern changes to focal ring with one dark hollow focus at its center. Increasing beam parameter  $\mu$  continuously broadens the intensity distribution in the focal region along the optical axis and the radius of the intensity ring in geometrical plane also increases, as shown in Fig. 4(e - h). From all the above evolution process, it can be seen that the beam parameter  $\mu$  affects the intensity distribution

remarkably in the focal region of the focused radially polarized QBG beam, and some novel focal patterns can come into being, which may be useful in constructing optical tweezers and carry out micro-manipulation.

#### 4. CONCLUSION

Focusing properties of the radially polarized QBG beam is investigated theoretically by vector diffraction theory. Result shows that intensity distribution in focal region of radially polarized QBG beam can be altered considerably by beam parameter  $\mu$  and radial parameter  $C$ . For instance of  $\mu = 0$ , it generates one focal spot, on increasing  $\mu$  can evolve from one focal spot to one focal ring, and the focal ring radius increases further increasing beam parameter and generates dark hallow focus. When changing radial parameter  $C$  focal shift phenomenon occurs along the optical axis, this is highly used for optical trapping applications and optical manipulation technique.

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