



# Tightly Focusing Properties of Radially Polarized Double Ring Shaped Beam through a Uniaxial Birefringent Crystal

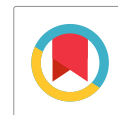
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## ABSTRACT

Using vectorial diffraction theory for small birefringence, the properties of a radially polarised Double ring-shaped beam closely oriented through a uniaxial birefringent crystal is computed. Increase the birefringence value  $[\Delta n]$  to adjust the intensity distribution of focal structure in the focal area around the longitudinal axis, and change the pupil to beam ratio  $[\beta]$  to separate the focal position. Also, by changing the pupil to beam ratio, a single focal spot can be broken into two.

**Keywords:** Radially Polarized Double Ring Shaped Beam; Uniaxial Birefringent Crystal.

## 1. INTRODUCTION

Tight focusing of the laser beam has attracted much attention because of its much smaller focus spot and strong longitudinal component (Urbach and Pereira, 2008; Dorn *et al.* 2003; Yan *et al.* 2011). In a range of microscopes and spectrosopes, tightly focused light beams can improve imaging precision and boost the concentrated electric field at the nanometer scale. Tightly focused light beams have wide potential applications in optical data storage, optical trapping, and microscopy (Zhang *et al.* 2008; Youngworth and Brown, 2000; Lerman and Levy, 2007; Walker and Milster, 2001; Helseth, 2001). The longitudinal field component at the focal point of a radially polarized beam (RPB) is larger than that of any other focused field (Urbach and Pereira, 2008). In 2003, direct detection and characterization of this sharp longitudinal field by an experimental demonstration was reported (Dorn *et al.* 2003). Tightly focused radially polarized beam has many attractive applications such as particle acceleration (Romea and Kimura, 1990), fluorescent (Novotny *et al.* 2003), second harmonic generation (Yew and Sheppard, 2007), Raman spectroscopy (Hayazawa *et al.* 2004), optical trapping (Zhan, 2004) material processing (Niziev and Nesterov, 1999) and particle acceleration (Tidwell *et al.* 1993), optical data storage (Xiangping *et al.* 2007), applications in high-resolution microscopy (Cheng *et al.* 2011), surface plasmon excitation (Zhan, 2006), particles accelerating (Varin and Piche, 2002) and in material processing (Meier *et al.* 2007). Normally the focusing beam was assumed to be a single-ring-shaped beam, which is often referred to as a radially polarized TEM<sub>01</sub>\* (R-TEM<sub>01</sub>\*) mode beam (Dorn *et al.* 2003; Youngworth and Brown, 2000; Quabis *et al.* 2000). On the other hand, a double-ring-shaped beam was experimentally observed as a higher-order radially polarized mode (R-TEM<sub>11</sub>\*)

directly from a laser cavity (Moser *et al.* 2005). By disruptive interaction between the inner and outer rings with a pi phase change, radially polarised mode (R-TEM<sub>11</sub>\*) beams will effectively decrease the spot size (Kozawa and Sato, 2006; 2007). Recently modulating the focal patterns using double ring-shaped beam is a topic of great interest, and many works are reported (Nie *et al.* 2014; Rajesh *et al.* 2011; Tian and Pu, 2014; Lalithambigai *et al.* 2013; Zhang *et al.* 2009; 2010). Many optical components, including waveplates, compensators, polarizer, and so on, make use of the birefringent effect of uniaxial crystals to realize their functions. Thus, it is of practical significance to study the propagation dynamics of light beams in uniaxial crystals. The properties of light beams propagating in birefringent materials have attracted many researchers' interest for years (Fleck and Feit, 1983; Stallinga, 2001; Ciattoni *et al.* 2002; Avendano-Alejo and Rosete-Aguilar, 2006). Ciattoni *et al.* have been studying the propagation of a paraxial optical beam along the optical axis of a uniaxial crystal in a series of papers (Ciattoni *et al.* 2002; 2003; 2001a; 2001b; 2001c; 2002a; 2002b; 2002c; 2002d; Provenziani *et al.* 2002; Ciattoni *et al.* 2002; 2003). Starnes *et al.* presented consistent numerical and experimental results for focusing two/three-dimensional electromagnetic waves into uniaxial crystal (Jain *et al.* 2009; Starnes and Jiang, 1998; Jiang and Starnes, 2000). The propagation of various kinds of laser beams in uniaxial crystals parallel and orthogonal to the optical axis has been reported (Lu and Luo, 2004; Deng *et al.* 2008; Tang, 2009; Li *et al.* 2010; 2011; Zhou *et al.* 2012; Deng *et al.* 2007; Liu and Zhou, 2008; 2009a; 2009b; 2009c; Du and Zhao, 2010; Zhang and Cai, 2011; Li and Chen, 2012; Zhou *et al.* 2013; Deng *et al.* 2013; Shen *et al.* 2014; Zhang and Cai, 2011). The tight focusing of radially, azimuthally and circularly polarized laser beams through a uniaxial crystal associated with focal shift have

received a lot of attention because of the theoretical and experimental interest (Zhang *et al.* 2008; Yonezawa *et al.* 2008; Rao and Wang, 2008). In this paper, based on the theoretical model of Ref. (Zhang *et al.* 2008; Rao and Wang, 2008), We use a uniaxial birefringent crystal to expand the study of tight focusing properties of radially

polarised double ring-shaped beams. It is shown that the intensity distribution of focal structure in the focal region can be shift along the longitudinal axis by increasing the birefringence value  $[\Delta n]$ , and splitting of focal spot can be obtained by changing the pupil to beam ratio $[\beta]$ .

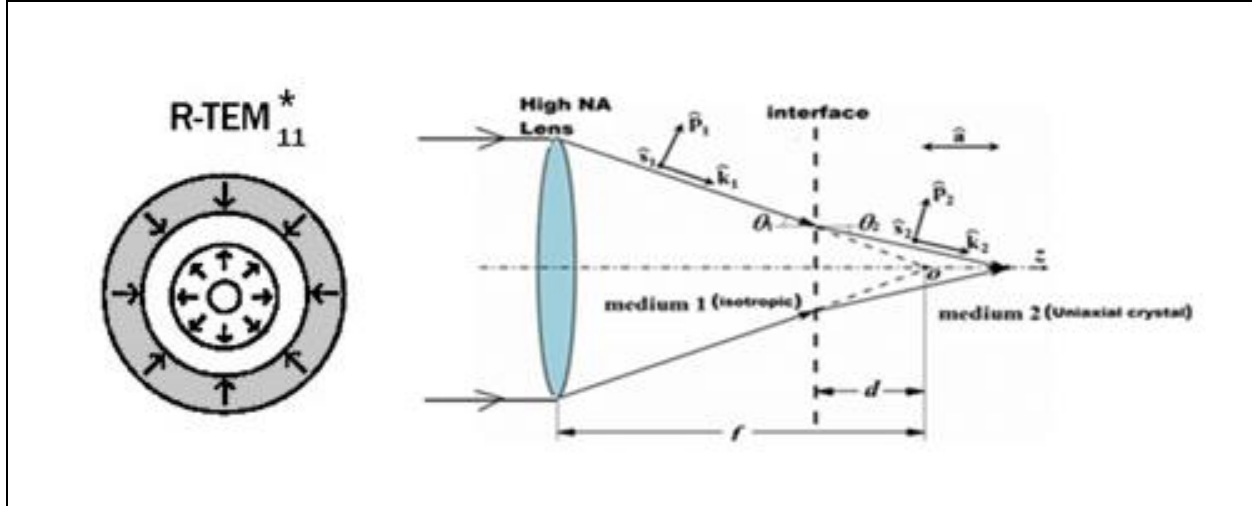


Fig. 1: Scheme of the optical system.

The schematic diagram of the focusing system is shown in Figure 1. It is thought that the radially polarised double ring-shaped beams concentrate from medium 1 through medium 2 (see Fig. 1). Medium 1 is isotropic, while medium 2 is a uniaxial birefringent with a symmetrical axis that runs parallel to the optical axis. Here  $d$  is the probe depth which is the distance between the interface and geometrical focus (Torok *et al.* 1995).  $\hat{k}_1$  and  $\hat{k}_2$  are the wave vectors in medium 1 and medium 2 with  $\hat{s}_1 \hat{p}_1$  and  $\hat{s}_2 \hat{p}_2$  are the corresponding polarization vectors in a parallel and perpendicular direction to the plane of incidence. Based on vectorial Debye theory (Gu, 2000), Cartesian components of the electric field vector in the focal region can be expressed as

$$E_{tot} = E_x(r, \psi, z) + E_y(r, \psi, z) + E_z(r, \psi, z) \rightarrow 1$$

Where

$$E_x(r, \psi, z) = \frac{-iE_o}{\pi} \int_0^\alpha \int_0^{2\pi} \sin \theta_1 \sqrt{\cos \theta_1} P(\theta_1) \exp[ik_2 z \cos \theta_2 + ik_1 r \sin \theta_1 \cos(\psi - \phi)]$$

$$[t_p \cos \theta_2 \cos \phi \exp[ik_o(W + \Delta W)]] \exp(-i\phi) d\phi d\theta_1 \rightarrow 2$$

$$E_y(r, \psi, z) = \frac{-iE_o}{\pi} \int_0^\alpha \int_0^{2\pi} \sin \theta_1 \sqrt{\cos \theta_1} P(\theta_1) \exp[ik_2 z \cos \theta_2 + ik_1 r \sin \theta_1 \cos(\psi - \phi)]$$

$$[t_p \cos \theta_2 \sin \phi \exp[ik_o(W + \Delta W)]] \exp(-i\phi) d\phi d\theta_1 \rightarrow 3$$

and

$$E_z(r, \psi, z) = \frac{-iE_o}{\pi} \int_0^\alpha \int_0^{2\pi} \sin \theta_1 \sqrt{\cos \theta_1} P(\theta_1) \exp[ik_2 z \cos \theta_2 + ik_1 r \sin \theta_1 \cos(\psi - \phi)]$$

$$[-t_p \sin \theta_2 \exp[ik_o(W + \Delta W)]] \exp(-i\phi) d\phi d\theta_1 \rightarrow 4$$

Here  $E_o$  is a constant related to the focal length and the wavelength,  $\alpha = \sin^{-1}(NA)$  is the maximal angle determined by the NA of the objective;  $t_p$  is the amplitude transmission coefficient for parallel polarization state, which is given by the Fresnel equations (Born and Wolf, 1999)

$$t_p = \frac{2 \sin \theta_2 \cos \theta_1}{\sin(\theta_1 + \theta_2) \cos(\theta_1 - \theta_2)} \rightarrow 5$$

$W_p = W + \Delta W$  and  $W_s = W$  are the aberration functions of p- and s-polarizations respectively. Here  $W$  is the aberration function caused by the mismatch of the refractive indices medium 1 and medium 2, where  $\Delta W$  is the phase difference between the ordinary and extraordinary modes in the uniaxial birefringent medium 2.  $W$  and  $\Delta W$  are expressed as (Stallinga, 2001).

$$W = kd(n_2 \cos \theta_2 - n_1 \cos \theta_1) \rightarrow 6$$

$$\Delta W = k(d + z)\Delta n \sin^2 \theta_2 / \cos \theta_2 \rightarrow 7$$

Where  $k = 2\pi/\lambda$  is the wavenumber in vacuum and  $d$  is the distance between the interface and the geometric focus; Where  $\Delta n = n_e - n_o$  represents the difference between the refractive indices of ordinary and extraordinary modes in the medium 2, which is the so-called birefringence (the ordinary and extraordinary refractive indices are  $n_o$  and  $n_e$  respectively, and  $n_o = n_2$ ). It is assumed that the focusing lens is corrected for aberrations introduced by an anisotropic cover layer of thickness  $d$  and refractive index  $n_2 = n_o$

As a result,  $W_s = W = 0$   $W_p = \Delta W$ .

Here  $P(\theta)$  is the pupil function of the incident double ring-shaped LG (1, 1) beam and is given by (Kozawa and Sato, 2006)

$$P(\theta) = \frac{\beta_o^2 \sin(\theta)}{\sin^2 \alpha} \exp\left(-\frac{\beta_o^2 \sin^2(\theta)}{\sin^2 \alpha}\right) L_p^1\left(2\frac{\beta_o^2 \sin^2(\theta)}{\sin^2 \alpha}\right) \rightarrow 8$$

Where  $\beta$ , is the ratio of the pupil radius to the incident beam radius in front of the focusing lens and  $L_p^1$  is the generalized Laguerre polynomial. Note that it  $\beta$  should be greater than 1 because the outer ring of the R-TEM11\* beam will be completely truncated by the pupil if  $\beta \leq 1$ . The focal properties are evaluated numerically for the incident radially polarized double ring-shaped beam by solving the above equations using the parameters  $NA = 0.85, n_1 = 1, n_2 = 1.5, \lambda = 400 \text{ nm}, d = 100 \mu\text{m}$ .

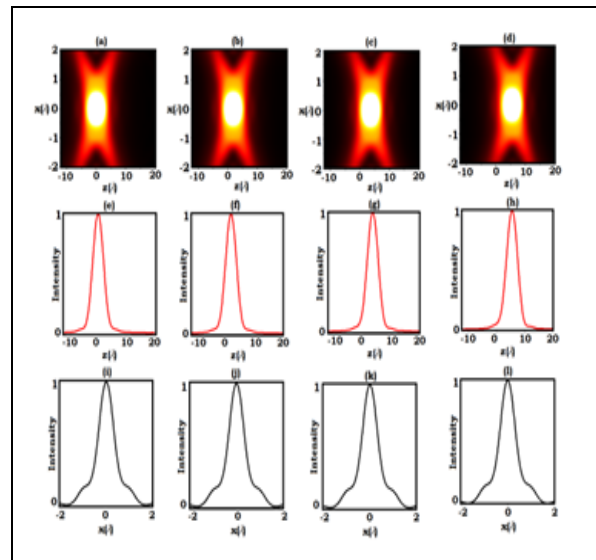
## 2. RESULT & DISCUSSION

Fig. 2(a-d) shows the intensity distribution in the x-z plane for the incident double ring-shaped beam with  $\beta = 1.1$  and different axial birefringence value ( $\Delta n$ ). Fig. 2(e-h) shows the corresponding axial intensity distribution, and Fig. 2(i-l) shows the lateral intensity distribution calculated at the point of maximum axial intensity. It is observed from Fig 2(a) and their corresponding radial and axial intensity distribution that the generated focal segment for  $\beta = 1.1$  and  $\Delta n = 0$  is a focal spot having FWHM of  $0.79 \lambda$  and focal depth of  $4.48 \lambda$ . However, we observed that increasing the axial birefringence to  $\Delta n$  to  $5, 10, 10^{-3}$  and shifts the focal segment in the axial direction and the corresponding on axial maximum is found to located at  $z = 1.8 \lambda, 3.7 \lambda, 5.5 \lambda$  respectively. We also noted that increasing the axial

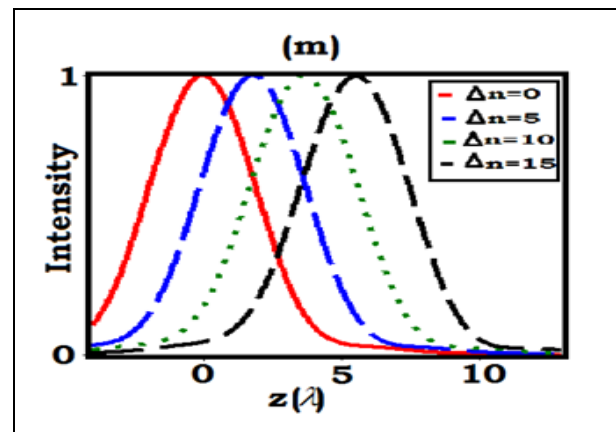
birefringence does not change the focal structure but generates only the axial shifting in the positive z-axis.

**Table 1. Showing the FWHM, depth of focus and focal shift for different  $\Delta n$  and for  $\beta = 1.1$ .**

| Beam Parameter $\beta = 1.1$      |                         |                                 |   |
|-----------------------------------|-------------------------|---------------------------------|---|
| Birefringence $\Delta n(10^{-3})$ | Spot size ( $\lambda$ ) | Depth of Focusing ( $\lambda$ ) | Position of Maximum Intensity Shifted ( $\lambda$ ) |
| 0                                 | 0.79                    | 4.48                            | 0   |
| 5                                 | 0.79                    | 4.48                            | 1.8   |
| 10                                | 0.79                    | 4.48                            | 3.7   |
| 15                                | 0.79                    | 4.48                            | 5.5   |



**Fig. 2: (a-d) 3D Intensity distribution in the x-z plane corresponding to  $\beta = 1.1$  and for  $\Delta n = 0, 5, 10, 15$ . (e-h) are the corresponding axial intensity distribution. (i-l) are the intensity distribution in the transverse direction measured at the point of maximum axial intensity.**



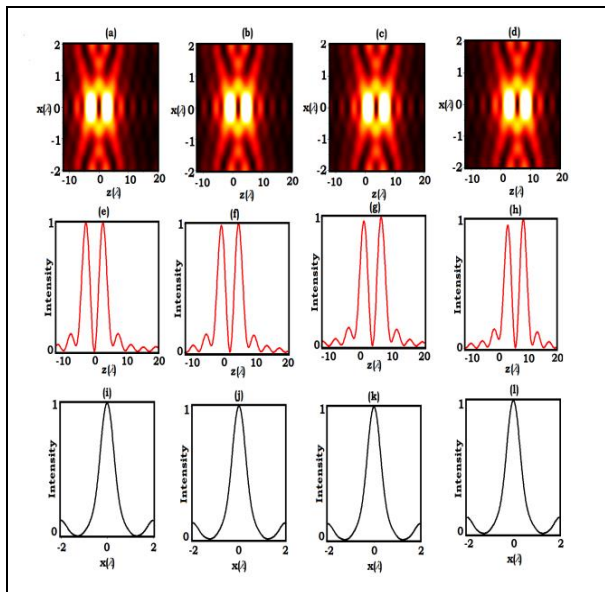
**Fig. 3(m): shows focal shift in the 2D on axial intensity distribution corresponding to different  $\Delta n$  values.**

Table 1 shows the FWHM, depth of focus, and maximum position on axial intensity obtained for different values of axial birefringence. To produce a focal shift profile, one may insert a physical mask such as a cosine wave plate in the pupil plane of the objective (Prabakaran *et al.* 2013; Li *et al.* 2010; Gao *et al.* 2016; 2009; Yun *et al.* 2010; Yan *et al.* 2011). However, the presence of a cosine wave plate or phase filter makes some applications more difficult or even impossible. To avoid these drawbacks here, we suggest a simple method of modulating the axial birefringence to the incident radially polarized double ring-shaped beam.

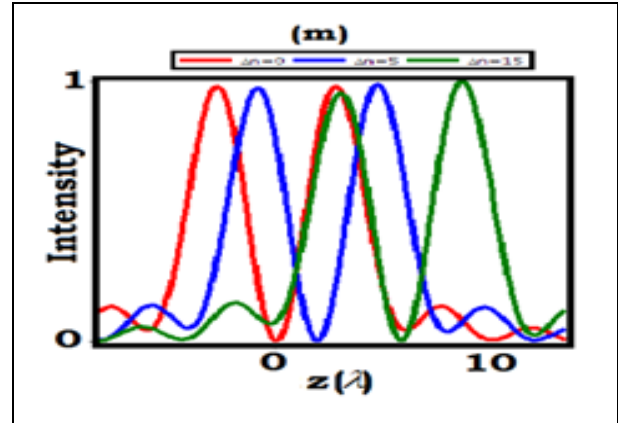
Fig. (3) shows the same as Fig. (2) but for  $\beta = 1.5$ . It is noted from Fig. 3(a) and their corresponding axial and transverse intensity distribution in the absence of axial birefringence. Increasing  $\beta$  to 1.5 generates an axially split focal segment, each having FWHM of  $0.74 \lambda$  and focal depth of  $2.80 \lambda$ . We observed that these spots are axially separated by a distance of  $(5.4)\lambda$ .

**Table 2. Showing the FWHM, depth of focus and focal shift for different  $\Delta n$  and for  $\beta=1.5$ .**

| Beam Parameter $\beta = 1.5$      |                    |                   |                           |
|-----------------------------------|--------------------|-------------------|---------------------------|
| Birefringence $\Delta n(10^{-3})$ | FWHM ( $\lambda$ ) | DOF ( $\lambda$ ) | Focal Shift ( $\lambda$ ) |
| 0                                 | 0.74               | 2.80              | 2.70                      |
| 5                                 | 0.74               | 2.80              | 4.50                      |
| 10                                | 0.74               | 2.80              | 6.40                      |
| 15                                | 0.74               | 2.80              | 8.30                      |



**Fig. 3: (a-d) 3D Intensity distribution in the x-z plane corresponding to  $\beta=1.5$  and for  $\Delta n = 0, 5, 10, 15$ . (e-h) are the corresponding 2D intensity measured in the axial direction. (i-l) are the 2D intensity distribution in the transverse direction.**



**Fig. 3(m): shows focal shift in the 2D on axial intensity distribution corresponding to different  $\Delta n$  values.**

It is observed from Fig. 3(b-d) and their corresponding transverse and axial intensity distribution plots that increasing axial birefringence shifts the generated focal segment in the positive axial direction without modulating the focal structure. Such a focal structure is useful in trapping and shifting two industrial particles (Gao *et al.* 2009; Qiufang Zhan *et al.* 2009). Hence it is noted that by properly manipulating the axial birefringence and pupil to beam ratio of the double ring-shaped beam, one can generate single and axially split focal spots and can shift axially.

### 3. CONCLUSION

Using vectorial diffraction theory for small birefringence, the properties of a radially polarised Double ring-shaped beam tightly oriented through a uniaxial birefringent crystal were analyzed. Increase the birefringence value [ $\Delta n$ ] to modify the intensity distribution of focal structure in the focal area around the longitudinal axis, and change the pupil to beam ratio [ $\beta$ ] to separate the focal position. Even, by changing the pupil to beam ratio [ $\beta$ ], a single focal spot can be broken into two.

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### CONFLICTS OF INTEREST

The authors declare that there is no conflict of interest.

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