Focusing of Radially Polarized Lorentz Gaussian Beam with One on Axis Optical Vortex

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Abstract

Focusing properties of radially polarized Lorentz–Gauss beam with one on-axis optical vortex was investigated by vector diffraction theory. Results show that intensity distribution in the focal region can be altered considerably by charge number of the optical vortex and the beam parameters. Many novel focal patterns may occur, Such as Peak-centered, and other focal shapes which is potentially useful in optical tweezers, material processing and laser printing.

***Keywords:** Lorentz gaussian beam; Radially polarized beam; Vector diffraction theory.*

1. INTRODUCTION

Recently, the Lorentz–Gauss beam has been introduced as a new kind of realizable beam (Gawhary *et al.* 2006). The Lorentz beam can be regarded as a special case of Lorentz–Gauss beams. With the spatial extension being the same, the angular spreading of a Lorentz–Gaussian distribution is higher than that of a Gaussian description (Naqwi *et al* 1990). Therefore, the Lorentz–Gauss beam provides an appropriate model to describe certain laser sources, e.g., double heterojunction Ga1"xAlxAs lasers, which produce highly divergent fields (Dumke *et al* 1975). Recently, a new type of optical beam called Lorentz-Gauss beam has attracted a great deal of interest. The existence of Lorentz-Gauss beam is demonstrated both in theory and in experiment. In theory, the Lorentz-Gauss beam is proved a closed-form solution of the paraxial wave equation (Bandres *et al.* 2007) in experiment, the

Lorentz-Gauss 'beam can be realized by certain double heterojunction lasers (Dumke *et al.* 1975 and Naqwi *et al*. 1990). The characteristics and applications of Lorentz-Gauss beams have been investigated (Torre *et al*. 2008, Zhou in 2008, Zhou in 2009, Zhou in 2010 and Zhou *et al*. 2010). Since then, the Lorentz–Gauss beams have been studied extensively. The vectorial structures of Lorentz–Gauss beams have been examined in the far field (Zhou in 2008). The analytical propagation expressions of Lorentz–Gauss beams beyond the paraxial approximation have been derived (Zhou in 2009 and Yu *et al.* 2010). The fractional Fourier transform has been applied to treat the propagation of the Lorentz–Gauss beam (Zhou in 2009). The focal shift of a Lorentz–Gauss beam focused by an aperture-lens system has been numerically investigated. Based on the second-order moments, the beam propagation factors of Lorentz–Gauss beams have been investigated (Zhou in 2009). The average intensity and spreading of a Lorentz–Gauss beam in turbulent atmosphere and propagation of a partially coherent Lorentz–Gauss beam through a

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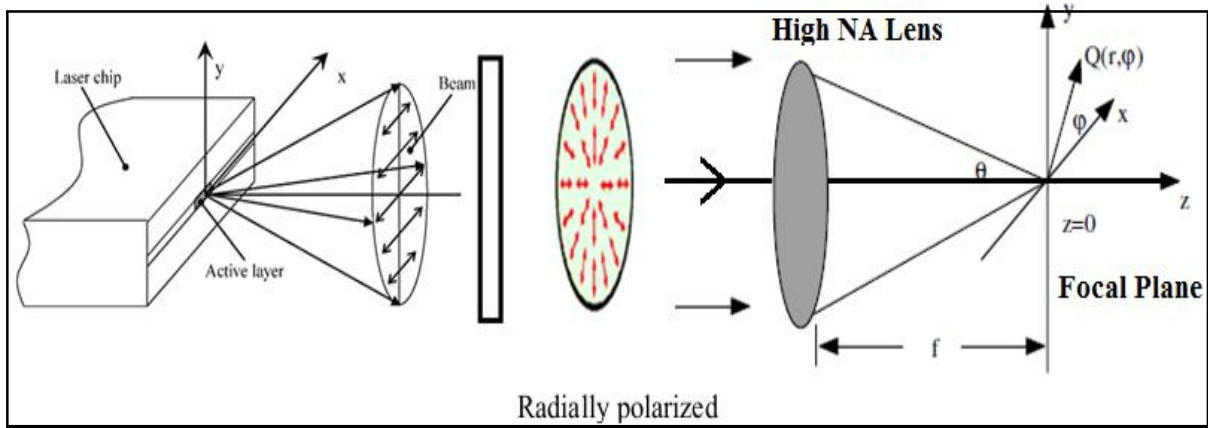*e-mail : rajeskb@gmail.com*

Fig.1. The sketch of the optical system, in which the Lorentz-Gauss beam is highly focused by the lens.

paraxial ABCD optical system were also investigated (Zhou in 2010). The propagation of Lorentz–Gauss beams in uniaxial crystals orthogonal to the optical axis and through an apertured fractional Fourier transform optical system were also studied (Zhao *et al.* 2010, Du *et al.* 2011). In addition, Radiation force of highly focused Lorentz–Gauss beams on a Rayleigh particle was also investigated theoretically (Jiang *et al.* 2011). However, to the best of our knowledge, the focusing of radially polarized Lorentz–Gauss beams containing optical vortex has not been studied so far. In order to get deep insight into the properties of Lorentz–Gauss beams and extend their applications, focusing properties of radially polarized Lorentz–Gaussian beam with one on-axis optical vortex was investigated by vector diffraction theory.

2. THEORY

A schematic diagram of the suggested method is shown in Fig. (1). The incident radially polarized Lorentz gaussian beam is focused through a high NA lens system. The analysis was performed on the basis of Richards and Wolf's vectorial diffraction method (Richards *et al.* 1959) widely used for high-NA lens system at arbitrary incident polarization. In the case of the incident polarization, adopting the cylindrical coordinates r,z,φ and the notations of Ref.

(Youngworth *et al.* 2005), radial and longitudinal components of the electric field $E_r(r,z)$ and $E_z(r,z)$ in the vicinity of the focal spot can be written as

$$\vec{E}\left(r,z\right) = E_r\vec{e}_r + E_z\vec{e}_z \rightarrow (1)$$

Where E_r, E_z are the amplitudes of the two orthogonal components and $\vec{e}_r, \vec{e}_z$ are their corresponding unit vectors. The two orthogonal components of the electric field is given as

$$Er = -iA/\pi \int_0^\alpha \int_0^{2\pi} \sqrt{\cos(\theta)} \times P(\theta) \times \sin(\theta)\cos(\theta)\cos(\phi-\varphi)\exp[ik(z\cos(\theta)$$
$$+r\sin(\theta)\cos(\phi-\varphi)]d\phi d\varphi \rightarrow (2)$$

$$Ez = iA/\pi \int_0^\alpha \int_0^{2\pi} \sqrt{\cos(\theta)} \times P(\theta) \times \sin(\theta)\cos(\theta)\sin(\phi-\varphi)\exp[ik(z\cos(\theta)$$
$$+r\sin(\theta)\cos(\phi-\varphi)]d\phi d\varphi \rightarrow (3)$$

Where $\alpha = \arcsin(NA)/n$ the maximal angle is determined by the numerical aperture of the objective lens, and n is the index of refraction between the lens and the sample. $k = 2\pi/\lambda$ is the wave number and $J_n(x)$ is the Bessel function of the first kind with order n. Here $P(\theta) = \exp(im\varphi)$ for a plane radially polarized vortex beam and for the Lorentz gaussian beam, the

$$P(\theta) = \exp\left[\frac{-\cos^2(\varphi) \times \sin 2(\theta)}{NA^2 W^2}\right] \times$$

$$\frac{1}{1 + \frac{\sin 2(\varphi)\sin^2(\theta)}{NA^2 \gamma^2}} \exp(im\varphi) \rightarrow (4)$$

electric field distribution can be written as (Fu Rui article in press) Hereis called relative beam waist, and can be called relative Lorentz parameter. ***m*** is the charge number of the optical vortex and W is called relative beam waist.

3. RESULT

We perform the integration of Eq. (1) numerically using parameters =1, and NA =0.95. Here, for simplicity, we assume that the refractive index n = 1. For all calculation in the length unit is normalized to λ and the energy density is normalized to unity. Fig.(2) illustrates the evolution of three-dimensional light intensity distribution of high NA lens for the incident plane radially polarized vortex beam and radially polarized Lorentz Gaussian beam based on eq.(1).
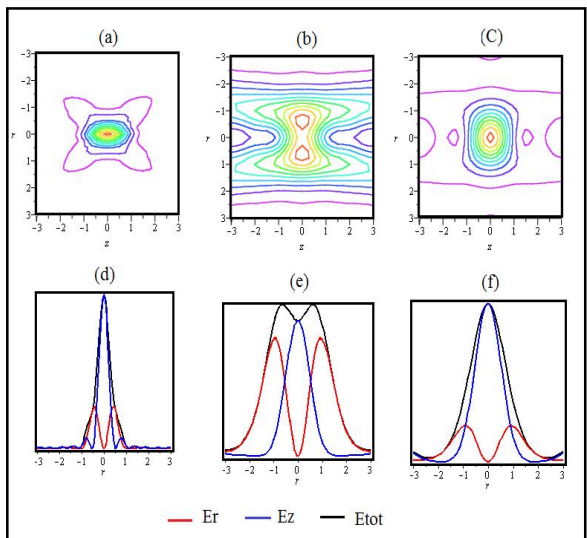

Fig. 2.(a & d) plane radially polarized beam, (b& e) radially polarized Lorentz gaussian beam for NA=0.95, W=0.3, =0.3, (c &f) radially polarized Lorentz gaussian beam for W=0.3, =1.2, and m=0

It is observed from Fig.2a, the generated focal segments for m=0 of a plane radially polarized beam is a focal spot having FWHM of 0.6λ and focal depth of 1.4λ. The 2D intensity profile calculated at the focus shown in Fig.2d revels that the longitudinal component of the intensity profile is dominating the radial component. Hence the generated spot is a longitudinally polarized one. Next we consider the tight focusing properties of radially polarized Lorentz Gaussian beam, with beam parameter W=0.3, γ= 0.3. Fig.2 b shows the focal segment generated by the tight focused Lorentz Gaussian beam of above mention beam parameters. It is observed from the Fig.2b, the focal segment of the generated focal spot extended radially with a bumpy structure of total intensity profile at the focus as shown in Fig.2e. It also observed, the radial and longitudinal components are almost equal in intensity and the FWHM of the generated focal spot is 2λ. Fig.2c & 2f shows the focal segment generated by the Lorentz Gaussian beam with beam parameter W= 0.3, γ = 1.2. It is observed that the generated focal segments is a focal spot with FWHM 1.74λ and focal depth of 1.32λ. The 2D intensity profile calculated at the focus, shows that the radial component is only 10% of the longitudinal component. Thus for the topological charge m=0, the plane radially polarized beam and the radially polarized Lorentz Gaussian beam with beam parameter (W= 0.3, γ = 0.3) shows similar behavior under the tight focusing condition except the fact that the focal spot generated by the Lorentz beam of above mentioned parameter is little bigger when compared to the plane radially polarized beam. However the Lorentz Gaussian beam with parameter (W= 0.3, γ = 1.2) shows completely different behavior due to the large radial component. Hence the behavior of the Lorentz Gaussian beam is very different from the plane radially polarized vortex beam of same topological charge.

4. CONCLUSION

Focusing properties of radially polarized Lorentz–Gauss beam containing one optical vortex was investigated numerically by the vector diffraction theory. Results show that the focal pattern can be

altered considerably by charge number of the optical vortex and the beam parameters. Many novel focal patterns may occur, such as peak-centered, is useful to the optical trapping, guiding, and manipulation of microscopic particles involved in the single mode diode laser beams.

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