



Tight Focusing Properties of Azimuthally Polarized Lorentz Gaussian Beam

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Abstract

The tight focusing properties of azimuthally polarized Lorentz Gauss beam is investigated theoretically by Vector Diffraction Theory. It is observed from the results that Non-Vortex Lorentz Gauss beam generated a sub wavelength focal hole under the tight focusing condition. It is also noted that FWHM of the focal hole and its focal depth suffers little change with the change in Lorential parameter. However when annular obstruction is introduced, the focal hole seems to get confined and improvement in the focal depth is observed. Focusing of Lorentz Gauss beam one optical vortex shows the formation of focal spot of sub wavelength size. It is also noted introduction of annular obstruction improved focal depth and reduced the spot size of the generated focal spot for the Lorential parameters considered.

Keywords: Azimuthally polarized; Lorentz Gauss beam; Vector diffraction theory.

1. INTRODUCTION

Recently, a new type of optical beam called Lorentz-Gauss beam has attracted a great deal of interest. The existence of Lorentz-Gauss beam is demonstrated both in theory and in experiment. In theory, the Lorentz-Gauss beam is proved a closed-form solution of the paraxial wave equation (Gawhary and Severini, 2006, Bandres and Gutiérrez-Vega, 2007) in experiment, the Lorentz-Gauss beam can be realized by certain double heterojunction lasers (Dumke, 1975; Naqwi and Durst, 1990). For instance, optical vortices can be used to construct highly versatile optical tweezers and arrays vortices can also assemble micro-particles into dynamically optical pumps (Naqwi and Durst, 1990). However, Gaussian approximation does not represent the field distribution perpendicular to the junction of a mono mode diode laser. The Lorentzian approximation is valid for a variety of commercially available double heterojunction (DH) Ga_{1-x}Al_xAs lasers, whose active regions are as narrow as 0.1 μm for a typical emission wavelength of 0.8 μm furthermore, the divergence of the field normal to the junction is generally so large that the beam is truncated (Gawhary and Severini, 2006). Recently, the experimental data from a double hetero structure laser were used to demonstrate that the far field distribution in the direction normal to the junction plane

approaches a Lorentzian function, and in the direction parallel to the junction it may be approximated by a Gaussian function (Dumke, 1975; Yu *et al.* 2010). They deduced the closed-form intensity distribution in the spatial frequency domain and investigated the effect of phase errors on the far-field intensity patent.

Recently, the Lorentz beams were also extended to the nonparaxial regime in which propagation properties is illustrated and compared with numerical examples. Li *et al.* (Zhou, 2009) derived analytically the properties and spreading of Lorentz beams through uniaxial crystal orthogonal to the optical axis by means of the theory of beam propagation in crystals and expansion method of Lorentz distributions. Zhou *et al.* (Du *et al.* 2011) applied the fractional Fourier transform to treat the propagation of Lorentz beams, and studied numerically propagation properties of a Lorentz beam in the fractional Fourier plane. Recently the tight focusing of linearly polarized Lorentz-Gauss beam with one optical vortex is recently studied by Fu Rui *et al.* and (Dawei Zhang *et al.* 2012; Xiumin Gao *et al.* 2013; Songlin Zhuang *et al.* 2012). Saraswathi *et al.* studied the tight focusing properties of radially polarized Lorentz Gauss beam beam (Saraswathi *et al.* 2014). In this article we studied the tight focusing properties of azimuthally polarized Lorentz Beam based on Vector Diffraction theory.

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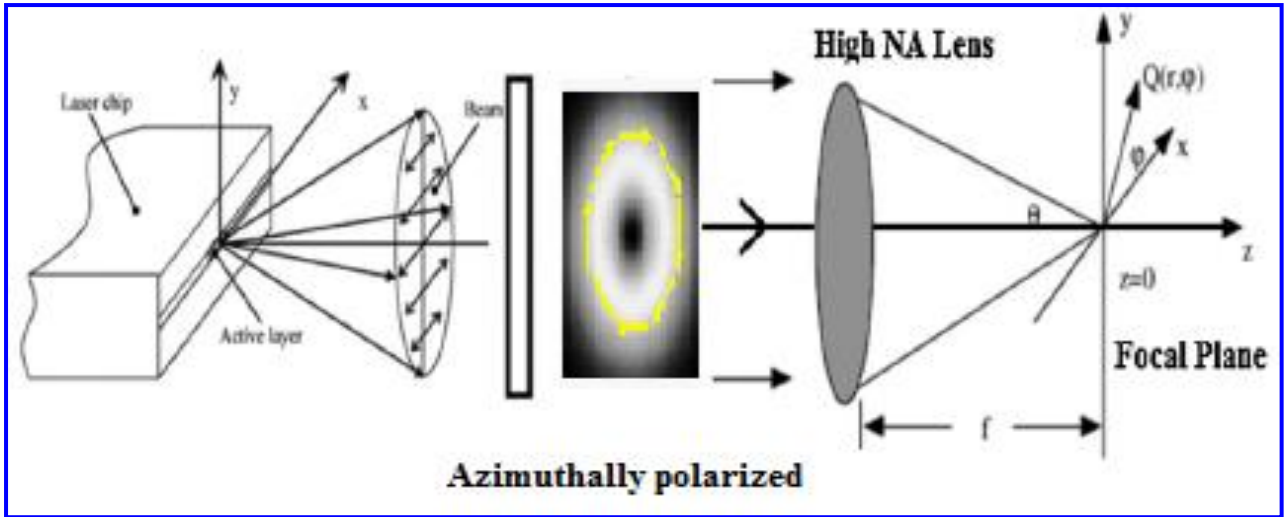


Fig. 1: The sketch of the optical system, in which the Lorentz–Gauss beam is highly focused by the lens.

2. THEORY

Azimuthally polarized Lorentz–Gaussian beam focused through a high NA lens system. The analysis was performed on the basis of Richards and Wolf’s vectorial diffraction method (Richards and Wolf, 1959) widely used for high-NA lens system at arbitrary incident polarization. In the case of the incident polarization, adopting the cylindrical coordinates r, z, φ and the notations (Youngworth and Brown, 2005), the electric field $E(r, z, \varphi)$ in the vicinity of the focal region can be written as

$$E(r, \varphi, z) = \begin{bmatrix} E_r \\ E_\varphi \\ E_z \end{bmatrix} = \begin{bmatrix} 0 \\ -\frac{iA}{\pi} \int_0^\alpha \int_0^{2\pi} \cos^{\frac{1}{2}}(\theta) \sin(\theta) A(\theta) \cos(\phi - \varphi) \exp(in\varphi) \exp[ik(z\cos\theta + r \sin\theta \cos(\phi - \varphi))] d\theta d\varphi \\ 0 \end{bmatrix} \rightarrow (1)$$

$n=0, 1, \dots$

Where $\alpha = \arcsin(NA)$, NA is the numerical aperture and n is the index of refraction between the lens and the sample. $A(\theta)$ Describes the Lorentz Gaussian beam, this function is given by (Xiumin Gao et al. 2013; Dawei Zhang et al. 2012)

$$A(\theta, \varphi) = \frac{C}{\omega_x \omega_y} \cdot \frac{1}{1 + \cos^2 \varphi (\sin^2 \theta / NA^2 \omega_x^2)} \times \frac{1}{1 + \sin^2 \varphi (\sin^2 \theta / NA^2 \omega_y^2)}$$

Where $\omega_x = \omega_x / r_p$ is called relative beam waist in x coordinate direction where r_p is the outer radius of optical aperture in focusing system, f is focal length of the focusing system. NA is the numerical aperture of the focusing system. C is chosen as constant in our investigations.

3. RESULTS

We perform the integration of eq. (1) numerically using parameters $\lambda = 1, NA = 0.95, n = 1$. Fig. 1 shows the focal structure generated for azimuthally polarized Lorentz beam for different Lorentzian parameter. It is observed from the Fig. 1(a), when $\omega_y = 0.3$ and $\omega_x = 0.3$, the generated focal segment is a focal hole having FWHM of 0.386λ and DOF of 1.72λ . However when we increase the Lorentzian parameter ω_y to 0.6, 0.9 and 1.2, the FWHM of focal hole slightly increased to 0.468λ with focal depth around 1.55λ is observed. These are shown in Fig. 2(b,c,d) and corresponding 2D plot in Fig. 2(f, g, h). The fig 2.(i,j,k,l) and their corresponding radially intensity shown in Fig. 2(m, n, o, p) shows the focal structure generated for the Lorentzian parameter $\omega_y = 0.3, 0.6, 0.9, 1.2$ when an annular obstruction with $\delta = 0.5$ is introduced at the objective. It is observed from the figure that the generated focal structure is a focal hole of FWHM of 0.33λ and focal around 1.951λ is observed. It is observed from the 2D plot the side lobe intensity corresponding to $\omega_y = 0.3$ is around 20%. Whereas, it decrease to 5%. when $\omega_y = 1.2$. Fig. 2(q,r,s,t) and 2D plot in Fig. 2(u,v,w,x) show the effect of annular obstruction with $\delta = 0.75$. It is observed from the figures the FWHM of the generated focal hole is around 0.298λ . where as its depth of focus is $3.406 \lambda, 3.13 \lambda, 3.622 \lambda$ and 3.69λ corresponding to $\omega_y = 0.3, 0.6, 0.9$ and 1.2 respectively. We also observed that the side lobe intensity is around 45% for $\omega_y = 0.3$, however it decreased to 5%. when we increase ω_y to 1.2. Thus on a whole tight focusing of the azimuthally polarized Lorentz beam generated a focal hole of almost same size and focal depth for all the Lorentzian parameter considered. However it is noted that focal hole size decreased to 0.298λ and 0.292λ and focal depth increased to 3.406λ and 3.69λ for all Lorentzian parameter when we use annular obstruction of $\delta = 0.5$ and $\delta = 0.75$ respectively. We also noted that side lobe intensity for the obstructed case is more for $\omega_y = 0.3$. However it gradually decreased when $\omega_y = 1.2$.

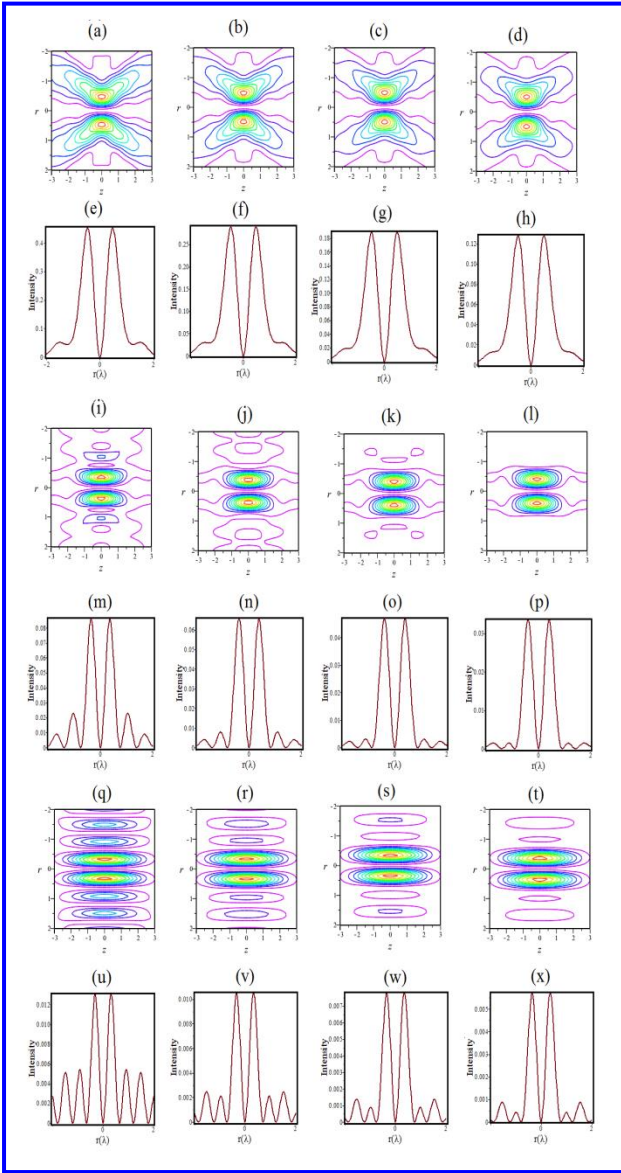


Fig. 2(a-x): Contour plot for the total intensity distribution in the r-z plane NA=0.95, n=0

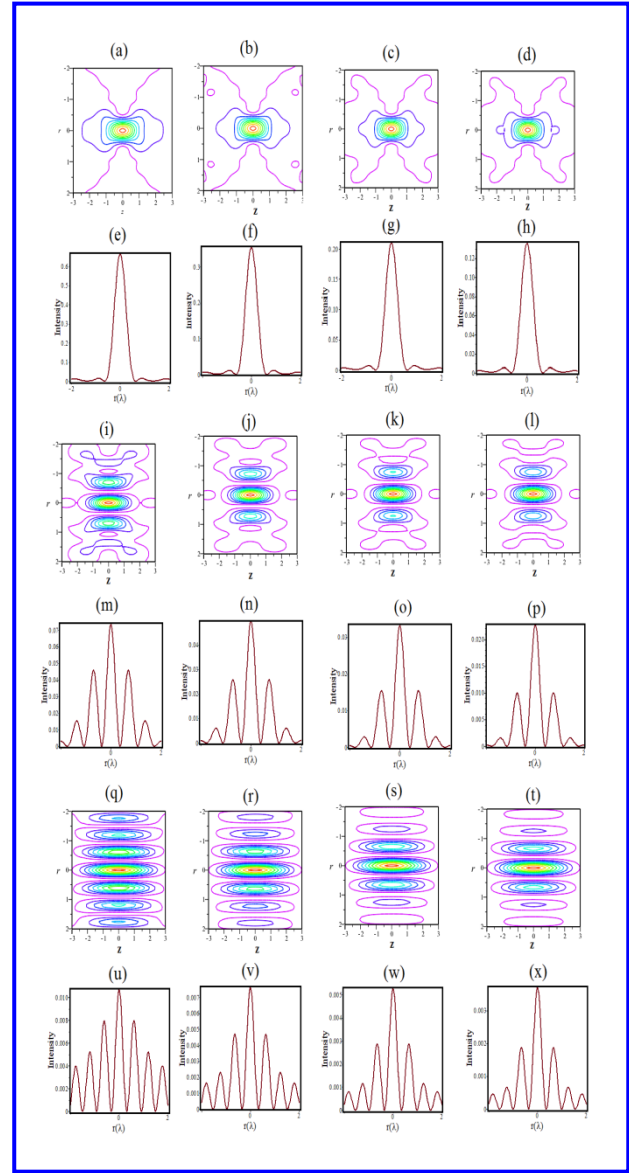


Fig. 3(a-x): Contour plot for the total intensity distribution in the r-z plane NA=0.95, n=1

Fig. 3 shows the same as Fig. 2 but for $n=1$. It is observed from the figure when $n=1$, the generated focal structure is a focal spot of FWHM around 0.504λ and focal depth of 1.8λ for all the Lorentz Parameter considered and are shown in Fig. 3(a,b,c,d) and its 2D plot in Fig. 3(e,f,g,h). It is also observed from Fig 3(i,j,k,l) and its corresponding 2D in Fig. 3(m,n,o,p) the FWHM of the focal spot almost reduced to 0.356λ and focal depth improved to 2.51λ for all the Lorentz parameter considered when an annular obstruction with $\delta=0.5$ is considered. It is also noted that side lobe intensity is maximum for $\omega_y=0.3$ as 75% where as it decreased to 10% when $\omega_y=1.2$. Fig. 3(q,r,s,t) and its 2D in Fig. 3(u,v,w,x) shows the effect of annular obstruction with $\delta=0.75$. It is observed from the figure that the generated focal spot has FWHM of 0.324λ and focal depth of 3.514λ . As it in the previous case it is noted that side lobe intensity is maximum for $\omega_y=0.3$ as 50% and minimum for $\omega_y=1.2$ as 5%.

4. CONCLUSION

The tight focusing properties of azimuthally polarized Lorentz Gauss beam is investigated theoretically by vector diffraction theory. It is observed from the result that Non-Vortex Lorentz Gauss beam generated the sub wavelength focal hole under the tight focusing condition it is noted that FWHM of the focal hole and its focal depth suffers little change with the change in Lorentz parameter. However when annular obstruction is introduced the focal hole seems to get confined and improve the focal depth is observed. Focusing of Lorentz Gauss beam one optical vortex shows the formation of focal spot of sub wavelength size. it is also noted introduction of annular obstruction improved focal depth and reduced the spot size of the generated focal spot for the Lorentz parameters considered. The authors expect such a high intense may find applications in optical, biological, and atmospheric sciences.

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CONFLICTS OF INTEREST

The authors declare that there is no conflict of interest.

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