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Investigation of Azimuthally Polarized Bessel-modulated Gaussian Beam with Annular Obstruction

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Abstract

Investigation of annular obstructed azimuthally polarized Bessel-modulated Gaussian beam (QBG) in the focal region of high NA lens, based on vectorial diffraction theory. The numerical results show that the intensity distribution in focal region of the incident beam can be altered considerably by changing beam parameter (μ) and introducing annular apodization (δ). Beam parameter induces the focal splitting in transverse direction, while annular apodization leads to change in focal pattern along optical axis of the focusing system. More interesting, the focal splitting may be in continuous in certain case of incident beam propagating through aligned optical system which is suitable for application such as optical manipulation and optical trapping.

Keywords: *Numerical aperture; focal splitting; focal hole; optical trapping.*

1. INTRODUCTION

The laser beams with cylindrical symmetry in polarization have attracted very much for their interesting properties and applications called cylindrical vector beams. This kinds of beams can be generated by using active and passive methods (Zhan *et al.* 2009; Youngworth *et al.* 2000; Gao *et al.* 2007; Zhou *et al.* 2007). Due to the polarization symmetry, the electric field at the focus has unique polarization properties. K.S. Youngworth and T.G. Brown calculated cylindrical-vector fields (Youngworth *et al.* 2000), which shows that, in the particular case of a tightly focused radially polarized beam, in the focal region the polarization shows large inhomogeneities, while the azimuthally polarized beam has purely transverse field in focal region. Focus shaping technique is also reported by using generalized cylindrical vector beams (Gao *et al.* 2007), in which a generalized cylindrical vector beam can be decomposed into radially polarized and azimuthally polarized components. And a generalized cylindrical beam can be generated from a radially polarized or an azimuthally polarized light using a two-half-wave-plate polarization rotator. Recently, the tight focusing properties of a polarized beam and a circularly, radially, azimuthally or linearly polarized vortex beam by a high numerical-aperture (NA) lens have been discussed (Quabis *et al*. 2001; Helseth *et al*. 2004; Grosjean *et al*. 2007; Ganic *et al*. 2003; Zhan *et al*. 2002; Helseth *et al*. 2006; Zhan *et al*. 2006). The study of coherent superposition of radially and azimuthally polarized components has also generated interest (Bokor *et al*. 2007). The polarization propagation and light intensity distribution in focal region plays an important role in many optical systems. It was demonstrated both theoretically and experimentally (Jia *et al*. 2005, Suresh *et al*. 2013). Optical intensity distributions in focal region have found applications in optical micro manipulation domain (Ashkin *et al*. 2003; Paterson *et al*. 2001; Gao *et al*. 2004; Caron *et al*. 1999; Hricha *et al*. 2005). Pure phase-shifting apodizer effect in focal region is reported (Gao *et al.* 2009). Nowadays, the Bessel-modulated Gaussian beams with quadratic radial dependence (QBG beam) have novel focusing properties in cylindrical coordinate system. Caron and Potvliege introduced QBG (Caron *et al.* 1999; Hricha *et al*. 2005; Wang *et al.* 2002; Belafhal *et al.* 2000; Lü *et al.* 2002), it has attracted much attention (Wang *et al*. 2003). The axisymmetric QBG beam are in zeroth-order, which is usually referred to as the axisymmetric QBG beam (Wang *et al*. 2008), Young worth and

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Brown calculated cylindrical-vector fields, near the focal region of an aplanatic lens, and briefly discussed some applications (Youngworth *et al.* 2000). Zhan and Leger reported a focus shaping technique using generalized cylindrical vector beam. In our knowledge, almost all QBG beams in previous papers are in scalar form without considering the polarization distribution of optical field. In this paper we numerically study the focusing properties of azimuthally polarized axisymmetric QBG beam with radial variance phase wavefront. Though the proposed method is simple and free from the complex interferometric methods used in the conventional techniques to shift the generated focal hole. The principle of the focusing radially polarized axisymmetric QBG beam with radial variance phase wavefront is given in Section 2. Section 3 shows the simulation results and discussions. The conclusions are summarized in Section 4.

2. THEORY

In the focusing system we investigated, focusing beam is azimuthally polarized axisymmetric QBG in radial (Youngworth *et al.* 2000; Zhan *et al.* 2002; Dorn *et al.* 2003; Suresh *et al.* 2014) Therefore the cylindrical coordinate system $(r,\phi,0)$ the field distribution $E(r,\phi,0)$ of the azimuthally polarized axisymmetric QBG beam at the plane z = 0 is written as,

$$E_0(r,\varphi,0)=E_0(r,\varphi,z=0)\cdot\left[\cos\left(\phi(r)\right)\cdot n_r\right] \quad (1)$$

Where $\phi(r)$ is the polarization angle from azimuthal direction, and is the function of azimuthal coordinate for the azimuthally polarized axisymmetric QBG beam, were n_r is azimuthal unit vector.

$$E_0(r,\varphi,z=0)=J_0\left(\frac{\mu r^2}{\omega_0^2}\right)\cdot\exp\left(\frac{r^2}{\omega_0^2}\right) \quad (2)$$

Where J_0 denotes the Bessel function of order zero, ω_0 is the waist width of the Gaussian beam, μ is a beam parameter which is complex valued in general. After some simple derivation (Tang *et al*. 2009), Eq. 2 can be rewritten as

$$E_0(r,\varphi,z=0)=J_0\left(\frac{\mu\sin^2(\theta)}{w^2\cdot NA^2}\right)\cdot\exp\left(\frac{\sin^2(\theta)}{w^2\cdot NA^2}\right)\exp\left(i\phi(r)\right) \quad (3)$$

Parameter $w = \omega_o/r_o$ is called relative waist width, where r_o is radius of incident optical aperture. NA is numerical aperture of the focusing system. θ is polar angle corresponding to radial coordinate. $\phi(r)$ is the polarization characteristics of the focusing azimuthally polarized axisymmetric QBG beam and can be expressed as

$$\phi(\theta)=C\cdot\frac{\tan(\theta)r}{\tan(\alpha)}\cdot\pi \quad (4)$$

Where $\alpha = \arcsin(NA/n)$ is the convergence angle of the focusing optical system. If the focusing optical system investigated is in air, i.e. refractive index n = 1. *C* is radial variance parameter indicating polarization radial degree the wave front phase distribution. Using the same analysis method as that in references (Youngworth *et al.* 2000; Gu *et al.* 2000) the electric field in focal region of azimuthally polarized axisymmetric QBG beam is,

$$E(r,\varphi,z)=E_r e_r+E_\varphi e_\varphi+E_z e_z \quad (5)$$

$$E_\varphi(r,\varphi,z)=2A\int_0^\alpha\sqrt{\cos(\theta)}\sin(\theta)E(\theta)J_1(kr\sin\theta)\exp(ikz\cos\theta)d\theta$$
$$E_r(r,\varphi,z)=E_z(r,\varphi,z)=0 \quad (6)$$

Where *A* is a constant, $J_o(x)$ and $J_1(x)$ denote zero order and first order Bessel functions of the kind, k is the wave number $k = 2\pi/\lambda$, with λ being the wavelength of illuminating beam. Were e_r, e_φ and e_z are unity vector.

Substitute Eq. (3) and (4) into Eqs. (6), then substitute Eqs. (6) into Eq. (5), we can obtain the analytical total electric field in focal region. The optical intensity in focal region is proportional to the modulus square of Eq. (5), so the focusing properties of azimuthally polarized axisymmetric QBG beam can be investigated theoretically.

3. RESULTS AND DISCUSSIONS

Without loss of validity and generality, it is proposed that the numerical aperture of the focusing system *NA = 0.95* and amplitude constant *A = 1*. The focusing properties of azimuthally polarized axisymmetric QBG beam is investigated theoretically, for parameter *C = 0* at different beam parameter *(μ)*. Firstly, the intensity distributions in focal region of the azimuthally polarized QBG beam are calculated under condition of *C = 0* and different *μ*, and are illustrated in Fig. 1. In this paper, it should be noted that the unit of coordinates in all figures in this article is k^{-1}. Focus refers to the maximum optical intensity peak, and focal shift denotes the movement of this focus.

Fig. 1 shows the total intensity distribution of incident azimuthally polarized QBG in the focal region of high NA lens (*NA = 0.95*). It can be seen from Fig. 1(a) in the focal region for *μ = 0*, i.e. the system generates two focal spot with equal intensities in radial direction forms focal hole of focal depth (1.4λ) each. The focus (focal hole) in the focal region splits along optical axis on increasing parameter μ, namely focal split phenomenon occurs and is shown in Fig. 1(b) for $\mu = 4$. On increasing $\mu = 6$ continuously, the focal pattern changes considerably and illustrated in Fig. 1(c). There is two bright spot with two weaker spot on either side of bright spot in the focal region. We further increased $\mu = 9$, in order to study the effect of beam parameter μ and its evolution in the focal region is shown in fig. 1 (d). We observed that on increasing μ the generated focal hole splits along optical axis and forms four bright spot in the focal region, the bright spot becomes weaker and the weaker spot becomes stronger.

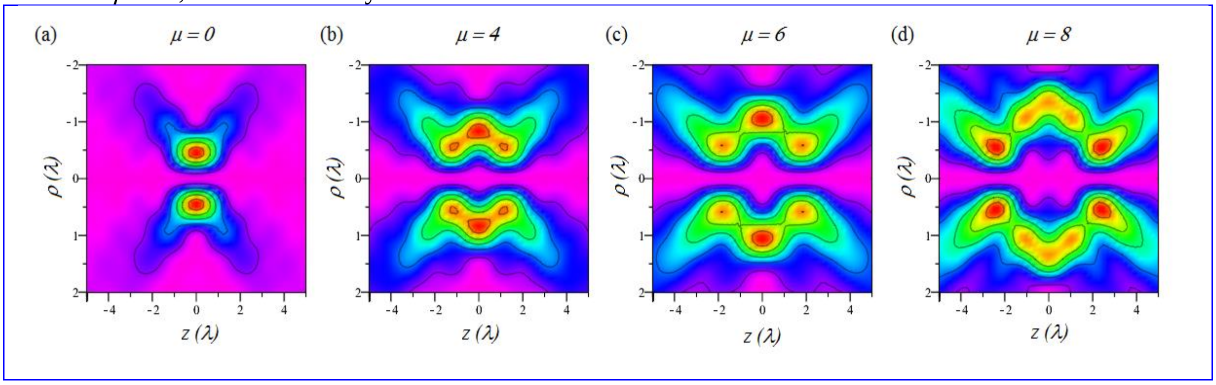

Fig. 1: Total intensity distribution of azimuthally polarized high NA lens (NA = 0.95) for (a) *μ=0*, (b) *μ=4*, (c) *μ=6* and (d) *μ=8*

Secondly, we analyzed numerically the effect of annular apodization for different parameter μ by keeping other parameter same as above. The effect of annular apodization on the incident radially polarized axisymmetric QBG beam is analyzed replacing *E(θ)* by *E(θ)T(θ) in* Eq. (5). Where *T(θ)* is given by

$$T(\theta) = \begin{cases} 0 & for \quad 0 \le \theta < \delta \cdot \alpha \\ 1 & for \quad \delta \cdot \alpha \le \theta < \alpha \end{cases} \tag{6}$$

In order to study the effect of annular apodization we choose annular apodizer $\delta = 0.85$, by keeping other parameter same as above for different pupil beam parameter and it is shown in Fig 2.

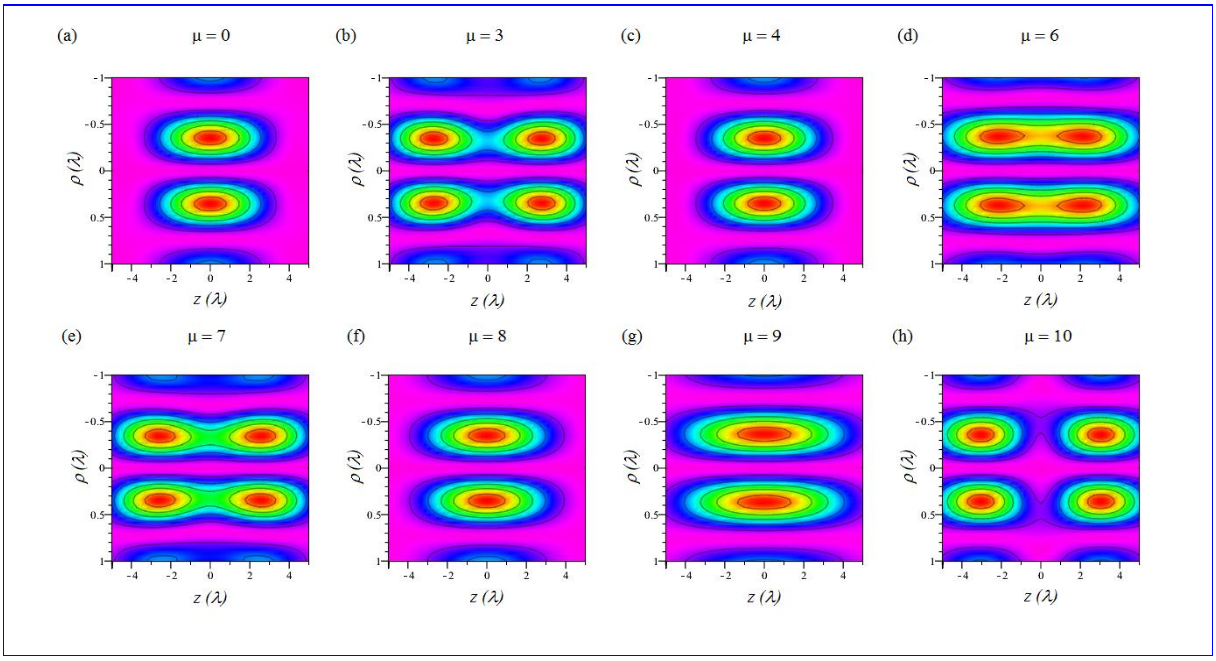

Fig. 2: Total intensity distribution of azimuthally polarized high NA lens (NA = 0.95) with annular obstruction (*δ = 0.85*) (a) *μ=0*, (b) *μ=3*, (c) *μ=4*, (d) *μ=6*, (e) *μ=7*, (f) *μ=8*, (g) *μ=9* and (h) *μ=10*

Fig. 2 (a) shows the total intensity distribution contour plot of incident annular obstructed azimuthally polarized axisymmetric QBG beam when μ = 0, the focal region has only one focal hole (two focal spot in radial direction with FWHM (0.28λ) and its focal depth of FWHM (4.06λ) of each). Further increase in beam parameter μ, results in focal splitting along axial direction at the focus and it is shown in Fig. 2 (b) for $\mu = 3$, for $\mu = 6$, generates focal hole in the focal region, it is shown in Fig 2 (c). Fig 2 (d) for $\mu = 6$, Fig. 2 (d) for $\mu = 7$, Fig. 2 (d) for $\mu = 8$, Fig. 2 (d) for $\mu = 9$ and Fig. 2 (d) for $\mu = 10$. It is observed from Fig. 2 on increasing μ continuously, the focal pattern changes from one focal hole to two focal hole in the focal region.

4. CONCLUSION

Based on vector diffraction theory, tight focusing of a azimuthally polarized QBG beam by a high NA lens is studied theoretically. The total optical intensity distribution in the focal region is illustrated by numerical calculations without and with annular apodization for different beam parameter μ. The simulation result shows, on increasing beam parameter μ the proposed system generates different focal pattern changes at the focus. We expect such a beam can be widely used in application such as optical trapping, data storage, biomedical imaging, laser drilling, and laser machining.

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CONFLICTS OF INTEREST

The authors declare that there is no conflict of interest.

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