



Tight Focusing of Double-ring-shaped Azimuthally Polarized Beam Using High NA Lens Axicon

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Abstract

The intensity distribution in the focal region for the double-ring-shaped azimuthally polarized beam using high NA lens axicon system is studied on the basis of the vector diffraction theory. A method is presented for generation of a sub wavelength (0.5λ) azimuthally polarized beam, which propagates without divergence over lengths of about (60.8λ) in free space. It is also observed that the distribution of the electric field does not change while the aberration coefficient of the lens axicon changes. The authors expect such a super-long dark channel may find applications in optical data storage, biomedical imaging, laser drilling and atmospheric sciences.

Keywords: Vector diffraction theory; Azimuthally polarized beam; High NA lens; High depth of focus.

1. INTRODUCTION

Generation of three-dimensional (3D) optical beams that are dark regions in space surrounded by light are driven by wide ranging applications including dark optical traps for atoms (Friedman et al. 2002, Veerabagu et al. 2013), manipulation, guiding and binding of micro-particles and biological cells (Cizmar et al. 2010), erase beams for super-resolution fluorescence microscopy (Watanabe et al. 2013), etc. Over the past years, a variety of techniques have been proposed for generating such optical bottle beams for applications in optical tweezers and atom traps (Arlt & Padgett 2000; Isenhower et al. 2009). A detailed framework to analyze the focused structure of an optical beam in high NA systems has been provided by Richards and Wolf (Richards & Wolf 1959). The polarization distribution of the beam can be ignored for the low NA systems in contrast to the high NA systems, where polarization state of the optical beam is taken into consideration in the diffraction integral. Recently, a sub wavelength focal hole (0.5λ) with a quite long depth of focus (26λ) was achieved near the focus by tight focusing of a double-ring-shaped azimuthally polarized beam with annular high Numerical Aperture (NA) annular lens (Tian & Pu 2011). Though the proposed method is simple and free from the complex

interferometric methods used in the conventional techniques to generate focal hole, it should be noted that a lot of energy of the incident light beam is blocked by the central part of the annular aperture.

Overcome this problem for the azimuthal polarization counterpart, a non-diffracting “dark channel” with a long DOF was more recently achieved by tight focusing of a double-ring-shaped azimuthally polarized beam with high NA lens axicon (Lalithambigai et al. 2012). The axicon lens and holographic axilens can generate long DOF (McLeod 1954; Davidson et al. 1991). However, the axial intensity is constrained by remarkable fluctuations due to unexpected diffraction. So such kind of optical beams also can be realized by placing the Diffractive Optical Element (DOE) on the lens pupil. Diffractive optical elements have been used to achieve long DOFs, and they are able to reduce the compromise between the NA and the DOF that determine the lateral and axial resolutions in a conventional optical imaging system, respectively. In this paper, we describe a numerical study, based on the vector diffraction theory, of a property of an azimuthally polarized double-ring-shaped A-TEM₁₁* mode beam that is tightly focused by a high NA lens axicon. The high NA lens axicon is a system of a cemented doublet-lens, where the virtual

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focal segment created by the aberrated diverging lens can be converted to a real focal segment, of the forward type with a nano scale resolution, by adding a high NA converging lens (Rajesh et al. 2010). We observe that our proposed system generates a sub wavelength 0.5λ focal hole with large uniform focal depth 60.8λ without any annular truncation. It is also observed that the distribution of the electric field does not change while the aberration coefficient of the lens axicon changes.

2. THEORY

In the case of the azimuthally incident polarization, adopting the cylindrical coordinates r, z, ϕ and the notations (Youngworth & Brown 2000), the electric field $E(r, \phi, z)$ in the vicinity of the focal region can be written as,

$$\vec{E}(r, z, \phi) = E_r \vec{e}_r + E_z \vec{e}_z + E_\phi \vec{e}_\phi \quad (1)$$

$$\begin{bmatrix} E_r \\ E_\phi \\ E_z \end{bmatrix} = \begin{bmatrix} 0 \\ 2A \int_0^\alpha \cos^2(\theta) \sin(\theta) A(\theta) J_1(kr \sin \theta) e^{i k z \cos \theta} d\theta \\ 0 \end{bmatrix} \quad (2)$$

Here A is relative amplitude, $\alpha = \arcsin(NA/n)$ is the maximum aperture angle with NA is the numerical aperture and n is the index of refraction between the lens and the sample. $J_1(x)$ denotes the Bessel functions of first kind, k is the wave number and the function $A(\theta)$ describes the amplitude modulation.

For illumination by a double-ring-shaped R-TEM₁₁* beam with its waist in the pupil, this function is given by (Youngworth & Brown 2000; Tian & Pu 2011)

$$A(\theta) = \beta^2 \frac{\sin \theta}{\sin^2 \alpha} \exp \left[- \left(\beta \frac{\sin \theta}{\sin \alpha} \right)^2 \right] L_p^1 \left[2 \left(\beta \frac{\sin \theta}{\sin \alpha} \right)^2 \right] \quad (3)$$

where β is the truncation parameter that denotes the ratio of pupil diameter to the beam diameter and L_p^1 is the generalized Laguerre polynomial. If $p = 1$, the incident azimuthally polarized beam is a tight focusing of double-ring-shaped azimuthally polarized beam. A schematic diagram of the suggested method is shown in Fig. 1. In the following simulation calculations, the unit of coordinates in all figures is the wavelength.

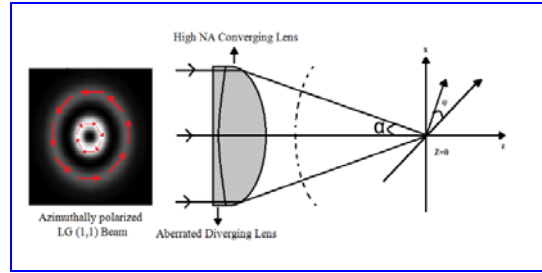


Fig. 1: Scheme for a double-ring-shaped azimuthally polarized beam focused by high NA lens axicon.

3. RESULTS AND DISCUSSION

In this article, first we describe a numerical study about the total electric field intensity distribution in the focal plane in the case of an ordinary lens is calculated, based on vector diffraction theory. We perform the integration of Eq. (1) numerically using parameters $\beta = 1.1$ and $\lambda = 1$. Here, for simplicity, we assume that the refractive index $n = 1$ and $A = 1$. Fig. 2(a) shows the contour plot of the total intensity distribution in the rz plane near the focus for the focusing system with high NA lens for $NA = 0.6$. It is observed that the DOF of the dark channel is about 9.6λ with Full Width Half Maximum (FWHM) is 0.862λ and the radii of the focal holes are almost uniform along the dark channel. We show it is possible to generate high intense sub wavelength focal hole with large uniform focal depth by using high NA lens axicon. The high NA lens axicon is a doublet of aberrated diverging lens and a high NA converging lens (Jaroszewicz & Morales 1998).

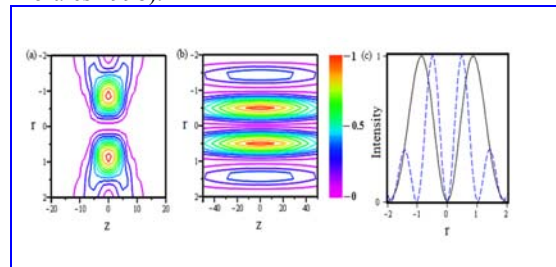


Fig. 2: Intensity distribution of the focused field near focus for the double-ring-shaped azimuthally polarized beam focused with (a) high NA lens; (b) high NA lens axicon; (c) the normalized intensity plot for lens (solid black line) and lens axicon (dashed blue line) for $NA = 0.6$.

Here we consider only systems that comprise a diverging lens that has third-order spherical aberration and a perfect high NA converging lens. The intensity distribution of the lens axicon is evaluated by

replacing the function $A(\theta)$ by the function $A(\theta) T(\theta)$ where $T(\theta)$ is the non-paraxial transmittance function of the thin aberrated diverging lens (Rajesh et al. 2010; Lalithambigai et al. 2012).

$$T(\theta) = \exp \left[ik \left(\psi \left(\frac{\sin(\theta)}{\sin\alpha} \right) \right)^4 + \left(\frac{1}{2f} \left(\frac{\sin(\theta)}{\sin\alpha} \right)^2 \right) \right] \rightarrow (4)$$

Fig. 2 (b) shows the contour plot of the total intensity distribution in the rz plane near the focus for the high NA lens axicon with NA = 0.6, respectively. The other parameters are same. It is observed that the radius of the focal holes are shown to be uniform along the dark channel and the dark channel have FWHM of total intensity around 0.5λ with corresponding focal depth is 60.8λ . The FWHM inferred from the normalized axial electric field intensity, as shown in Fig. 2(c) (dashed blue line), is approximately 0.5λ , which is 1.74 times smaller than that achieved by using high NA lens system. From Fig. 2(b), its DOF is very large and is almost 6.33 times greater than the high NA lens with equal NA. Therefore, it is possible to achieve a long DOF, high homogeneity, and a relatively small beam area simultaneously under this circumstance.

We also studied the total intensity distribution near the focus for different values of aberration coefficient ψ of the high NA lens axicon and are shown in Fig. 3. From the figure it is observed that the intensity distribution does not change while the aberration coefficient of the lens axicon changes from $2 \times 10^{-5} \text{ mm}^{-3}$ to $8 \times 10^{-5} \text{ mm}^{-3}$.

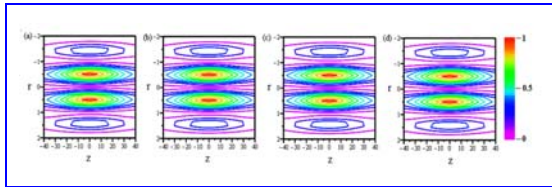


Fig. 3: Intensity profile of total field on focal plane of high NA lens axicon for A-TEM₁₁* beam and different values of spherical aberration coefficient of lens: (a) $2 \times 10^{-5} \text{ mm}^{-3}$ (b) $4 \times 10^{-5} \text{ mm}^{-3}$; (c) $6 \times 10^{-5} \text{ mm}^{-3}$; (d) $8 \times 10^{-5} \text{ mm}^{-3}$.

The main advantage of such a system is that it can generate a sub wavelength azimuthally polarized beam of uniform hole size and long focal depth even if there is a small change in the truncation parameter of the incident beam and the aberration coefficient of the lens axicon.

4. CONCLUSION

In conclusion, we have investigated the focusing properties of a double-ring-shaped azimuthally polarized beam through high NA lens axicon based on vector diffraction theory. We observed that the proposed high NA lens axicon can generate a sub wavelength super long dark channel with an FWHM of 0.5λ and long focal depth is around 60.8λ . The electric field does not change while the aberration coefficient of the lens axicon changes. The authors expect such a high intense super long dark channel may find applications in optical data storage, biomedical imaging, laser drilling and atmospheric sciences.

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