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Effect of Coma on Tightly Focused Linearly Polarized Lorentz Gaussian Beam

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Abstract

In this paper attention is given to the effects of primary coma on the linearly polarized Lorentz–Gauss beam with one on-axis optical vortex was investigated by vector diffraction theory. It is observed that by properly choosing the topological charge one can obtain many novel focal patterns suitable for optical tweezers, laser printing and material process. However, it is observed that the focusing objective with coma generates structural modification and positional shift of the generated focal structure.

Keywords: *Coma; optical trapping; Linearly polarized Lorentz Gaussian beam; Vector diffraction theory;*

1. INTRODUCTION

Vortex infected beams have generated considerable interest in many researchers. These type of beams can be experimentally generated by cooperative frequency locking in a helium–neon laser (Tamm 1988), through conversion of Hermite–Gaussian beams by an astigmatic mode converter (Abramochkin *et al.* 1991), by a spiral phase plate (Beijersbergen *et al.*1994) or through a computer generated hologram (Heckenberg *et al.*1992) . In recent years, propagation of vortex containing beam in free space or through aperture systems has drawn attention of (Indebetouw 1993). The propagation dynamics of optical vortices is influenced by phase and intensity gradients of the background beam, giving rise to a radial motion of the vortex in the direction of the transverse energy flow (Rozas *et al.* 1997). The gradients of the background beam act like driving forces in the motion of the vortex. The propagation of a Laguerre–Gaussian (LG) beam through a slightly misaligned paraxial optical system studied (Cai *et al.* 2006). Investigated the diffraction of various types of beams through spiral phase plates, and have presented theoretical and experimental results (Kotlyar *et al.* 2005). Helical structure of constant phase surface in the vortex containing beam makes it unique and different from other conventional beams. This unique feature is exploited in several applications ranging from optical trapping (Gahagan *et al.*1999) to optical testing (Senthilkumaran *et al.* 2003, Furhapter *et al.* 2005).

Recently, the Lorentz–Gauss beam has been introduced as a new kind of realizable beam (Gawhary *et al.* 2006). The Lorentz beam can be regarded as a special case of Lorentz–Gauss beams with the spatial extension being the same, the angular spreading of a Lorentz–Gaussian distribution is higher than that of a Gaussian description (Naqwi et al. 1990, Zhou in 2010, Saraswathi et al. 2013). Effect of aberrations on diffraction pattern in focal volume of optical systems has been an area of interest for a long time. (Biss *et al.* 2003) have investigated the effect of primary aberrations on the focused structure of the radially polarized vortex beam.(Wada *et al.* 2005) investigated the role of astigmatism, and comatic aberration on the propagation characteristics of a LG beam. There are several applications where size and shape of the dark core in the diffraction pattern of vortex carrying beam play an important role. Particle trapping, vortex mask as window for astronomical application, and optical processing are some of the examples. Recently (R.K.Singh *et al.* 2006) studied the role of astigmatism and coma on the diffraction pattern of a vortex containing beam with uniform amplitude distribution. However, to the best of our knowledge, the tight focusing properties of linearly polarized Lorentz–

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Gauss beam containing optical vortex in presence of coma has not been studied so far. The performance of the focusing system in the presence of primary coma is evaluated by using a vectorial Debye integral. In this paper we present the results of intensity distribution of linearly polarized Lorentz–Gauss beam with and without coma.

2. THEORY

In the focusing systems, the incident beam is Lorentz–Gauss beam, whose geometric parameters and coordinate system are shown in Fig. 1.

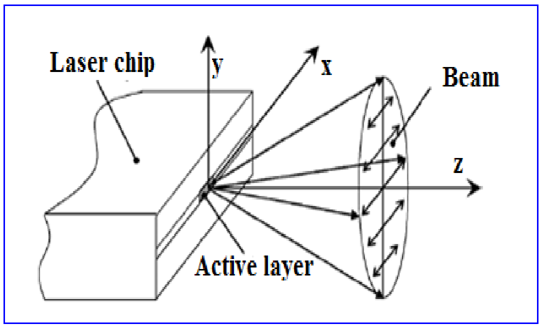

Fig. 1: Schematic of laser chip and rectangular coordinate system for laser diode

The amplitude distributions of the electric field in the directions parallel and normal to the junction are the Lorentzian and Gaussian functions, respectively (Naqwi *et al*. 1990, Dong *et al.*1993)

$$E_0(x_0,y_0)=E_0(x_0)E_0(y_0)=\exp\left(-\frac{x_0^{\,2}}{\omega_0^{\,2}}\right)\frac{\gamma_0^{\,2}}{\gamma_0^{\,2}+y_0}\rightarrow(1)$$

where parameters ω_0 and γ_0 are the 1/e -width of the Gaussian distribution and the half width of the Lorentzian distribution, respectively (Gawhary *et al.* 2006) . In order to calculate the focusing properties easily and clearly, the Eq. (1) can be rewritten as,

$$E_0(x_0,y_0)=\exp\left(-\frac{x_0}{\omega_0}\right)^2\frac{1}{1+(y_0/\gamma_0)^2}\rightarrow(2)$$

Now the radial coordinate in z = 0 plane is introduced, therefore,

$$x_0=r_0\cos(\varphi),y_0=r_0\sin(\varphi)\rightarrow(3)$$

Where r_0 is the radial coordinate, and ϕ is the azimuthal angle. When the incident Lorentz–Gauss beam contains one on-axis optical vortex, Eq. (1) can be rewritten as

$$E_0(r_0,\varphi)=\exp\left[-\cos^2(\varphi)\left(-\frac{r_0}{\omega_0}\right)^2\right]$$
$$\times\frac{1}{1+\sin^2(\varphi)\left(-\frac{r_0}{\gamma_0}\right)^2}\exp(im\varphi)\rightarrow(4)$$

Where, m is the charge number of the optical vortex, and the r_0/ω_0 can be written in the form,

$$\frac{r_0}{\omega_0}=\frac{r_0}{f}\frac{f}{r_p}\frac{r_p}{\omega_0}=\frac{\sin(\theta)}{NA\omega_x}\rightarrow(5)$$

Where $w_x = \omega_0/r_p$ is called relative beam waist in y coordinate direction and also called as relative Gauss parameter. r_p is the outer radius of optical aperture in focusing system, f is focal length of the focusing system. NA is the numerical aperture of the focusing system, defined as the multiple of refractive index of surrounding medium and sine value of aperture angle,

$$NA=n\sin(\alpha)=\frac{r_p}{f}\rightarrow(6)$$

n is refractive index of surrounding medium, the system we investigated is in air, so $n\approx1$, α is aperture angle. By similar deviation $\frac{r_0}{\gamma_0}$ can be written as

$$\frac{r_0}{\gamma_0}=\frac{r_0}{f}\frac{f}{r_p}\frac{r_p}{\gamma_0}=\frac{\sin(\theta)}{NA\gamma_\gamma}\rightarrow(7)$$

where $\gamma_\gamma - \gamma_0 / r_p$ is called relative beam waist in x coordinate direction, and can be called relative Lorentz parameter. Therefore, the electric field distribution can be rewritten as,

$$E_0(\theta,\varphi)=\exp\frac{\left[-\cos^2(\varphi)\times\sin 2(\theta)\right]}{NA^2\omega_x^{\,2}}$$
$$\times\frac{1}{1+\frac{\sin 2(\varphi)\sin^2(\theta)}{NA^2\gamma_\gamma^{\,2}}}\exp(im\varphi)\rightarrow(8)$$

It is assumed that the incident Lorentz–Gauss beam is linearly polarized along the x axis. According to vector diffraction theory, the electric field in focal region can be written in the follow form (Ganic *et al.*2003)

$$E(\rho,\psi,z)=\frac{1}{\lambda}\iint_{\Omega}\begin{Bmatrix}[\cos\theta+\sin 2\psi(1-\cos\theta]\times\cos\varphi\sin\varphi(\cos\theta-1)y\\ +\cos\varphi\sin\theta z\end{Bmatrix}\times\exp\frac{\left[-\cos^2(\varphi)\times\sin 2(\theta)\right]}{NA^2\omega_x^{\ 2}}$$

$$\times\frac{1}{1+\frac{\sin 2(\varphi)\sin^2(\theta)}{NA^2\gamma_\gamma^{\ 2}}}\exp(im\varphi)\times A_1\times\exp\left[-ik\rho\sin\theta\cos(\varphi-\psi)\right]\exp(-ikz\cos\theta)\sin\theta d\theta d\varphi\rightarrow(9)$$

where $\varphi\in[0,2\pi)$, $\theta\in[0,\arcsin(NA)]$. Vectors **x**, **y**, and **z** are the unit vectors in the x, y, and z directions, respectively. It is clear that the incident beams is depolarized and has three components (E_i, E_j and E_k) in **x**, **y**, and **z** directions, respectively. The variables ρ, ψ and z are the cylindrical coordinates of an observation point in focal region. The optical intensity in focal region is proportional to the modulus square of Eq. (9). A_1 denotes the wave front aberration function in the beam which can be expressed as (Kant 1995)

$$A_1=\exp\left[I.k.Ac\left(\frac{\sin(\theta)}{\sin(\alpha)}\right)^3\cos\phi\right]\rightarrow(10)$$

Where the coma coefficient Ac is in units of the wave length of the beam.

3. RESULTS

We perform the integration of Eq. (1) numerically using parameters λ=1, and NA =0.95. Here, for simplicity, we assume that the refractive index n = 1.For all calculation in the length unit is normalized to λ and the energy density is normalized to unity.

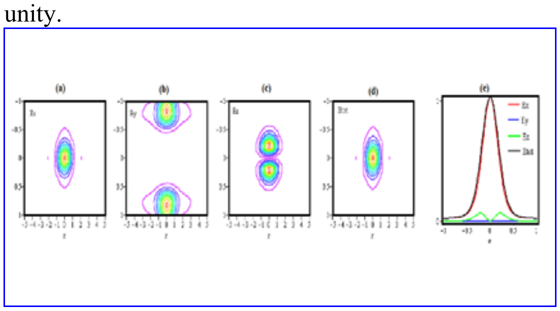

Fig. 2: Total intensity distribution of linearly polarized Lorentz Gaussian beam (a) Ex Component (b) Ey Component (c) Ez Component (d) Etot Component and fig.(e) Two dimensional intensity distribution at the focal plane for Ac=0,m=0

The three dimensional intensity distribution of a tightly focused Lorentz beam with m=0, ω=0.3 and γ=1.2,the fig 2(a) shows the x-component is a focal spot having FWHM 0.5 λ and focal depth of 1.6 λ. Fig 2(b) shows the y-component is a feeble intense focal spots is splited in the radial direction. The z- component shows in the fig 2(c) is a focal hole having FWHM 0.43 λ and focal depth of 2λ. The total intensity shown in fig 2(d) is a focal spot having FWHM 0.42 λ and depth of focus 1.9 λ. The fig.2(e) shows the two dimensional intensity distribution of component intensity at the focus. It is observed from the fig the x-component dominates the total intensity sector and the z-component is only 10% of the x-component.

The fig 3 shows the effect of coma for the above beam parameters. It is observed from the fig 3(a) the presence of coma with Ac=1λ for the x-component deformed its focal structure by extending the focus both radially and axially. The y-component showing in fig 3(b) is found to be shifted in the radial direction.

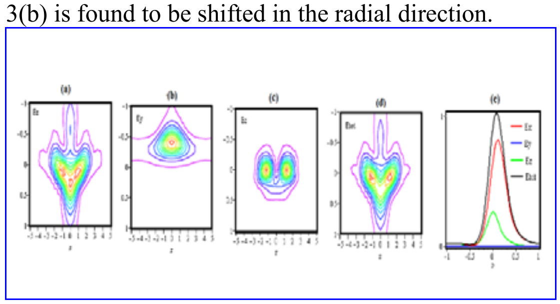

Fig. 3: Total intensity distribution of linearly polarized Lorentz Gaussian beam (a) Ex Component (b) Ey Component (c) Ez Component (d) Etot Component and fig.(e) Two dimensional intensity distribution at the focal plane for Ac=1λ, m=0

The z-component of intensity shown in fig 3(c), reveals that the generated focal structure is a focal spots. The total intensity shown in fig 3(d) resembles the structure of x-component. This is due to the fact of x-component plays the dominating role and is shown in fig 3(e).it is also observed from the fig the intensity of z-component is slightly increased when compared to the unaberrated case.

Fig 4(a) shows the intensity distribution for Ac=2λ. It is observed from the fig 4(a) increasing Ac splited the x-component in the axial direction with a central bright spot elongated much in the radial direction. The fig 4(b) shows the y-component also get deformed and splited into two focal spots with less intense central spot. The z-component shows in fig 4(c) is also found to be deformed and splited in the axial direction. The total intensity distribution shown in fig 4(d) resembles the x-component (dominating) i.e the focal segment splited unequally in the axial direction and elongated in the radial direction. The two dimensional component intensity calculated at the point of maximum intensity reveals slight increase in Z-component when compare to the unabebrrated case. Hence objective with coma strongly disturb the focal segment for Lorentz beam with and without vortex. The author expects the present study is useful in application when direct focusing of Lorentz beam emitted from the hetrojuntional laser diode.

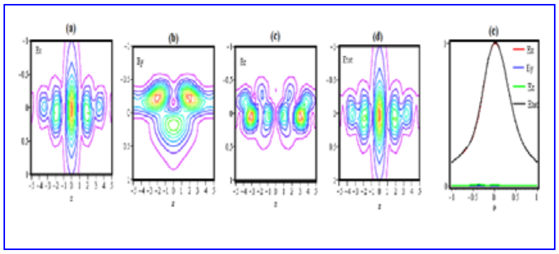

Fig. 4: Total intensity distribution of linearly polarized Lorentz Gaussian beam (a) Ex Component (b) Ey Component (c) Ez Component (d) Etot Component and fig.(e) Two dimensional intensity distribution at the focal plane for Ac=2λ, m=0

4. CONCLUSIONS

Effect of coma on the tight focusing properties of a linearly polarized Lorentz–Gauss beam containing one optical vortex was investigated numerically by the vector diffraction theory. Results show that the focal pattern can be altered considerably by charge number of the optical vortex and the beam parameters. The results show that the presence of coma generates positional shift, structural modifications and spreading of intensity distribution. The author expect such a study is important in practical applications such as optical tweezers, laser printing and material processing.

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