



## Focusing Properties of Radially Polarized Annular Multi Gaussian Beam by High NA Parabolic Mirror

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### Abstract

The focal properties of radially polarized annular multi Gaussian beams focused with a high numerical aperture parabolic mirror are numerically investigated. The tightly focused radially polarized annular multi gaussian beam beams by a high numerical aperture parabolic mirror have possible applications in particle acceleration, optical trapping and manipulating, single molecule imaging and high resolution imaging microscopy.

**Keywords:** Radially polarized beam; High NA Mirror; Vector diffraction theory.

### 1. INTRODUCTION

Recently, Radially polarized light has been gaining great attention due to its novel properties. Laser beams with radial polarization are characterized by a strong longitudinal electric field (Youngworth and Brown 2000) and a smaller spot size (Quabis *et al.* 2001) at the focal point, when the beams are tightly focused. The existence of a strong longitudinal field of tightly focused radially polarized beam has many attractive applications such as particle acceleration (Novotny *et al.* 2003), fluorescent imaging (Yew and Sheppard 2007), second harmonic generation (Novotny *et al.* 2003), and Raman spectroscopy (Hayazawa *et al.* 2004). It is reported that radial polarization to be the preferred approach for pupil masks for super resolution and apodization at high NA (Sheppard and Choudhury 2004, N. Veerabagu Suresh *et al.* 2013). Radially polarized beams with a circular  $\pi$  phase plate have also been used to produce a 3D optical cage ( Bokor and Davidson 2007). One is that the gradient forces formed by the radially polarized beams are much greater than those formed by the other two polarized beams. Because the stronger longitudinal intensity component of the tightly focusing radially polarized beam provides a stronger gradient force (Youngworth and Brown 2000), radially polarized beams have a higher axial

trapping efficiency than circularly polarized doughnut beams (Kawauchi *et al.* 1989). Specifically, for the aplanatic lens,  $A(\theta) = \cos^{1/2} \theta$  and for the parabolic mirror,  $A(\theta) = 2/(1 + \cos\theta)$ . It means that, due to energy conservation, in a parabolic mirror much of the incident energy reaches the focus under high angles while the contrary is valid for an objective lens. In this paper, the focusing properties of radially polarized annular multi gaussian beams by a high NA parabolic mirror are numerically investigated. The vector field distribution within the focal region and three-dimensional intensity distribution for radially polarized annular multi Gaussian beams are calculated by using the Richards–Wolf vectorial diffraction theory.

### 2. THEORY

At optical frequencies, only the electric field is responsible for the interaction with matter (scattering, fluorescence, excitation, polarization, etc.). The analysis was performed on the basis of Richards and Wolf's vectorial diffraction method widely used for high-NA mirror system at arbitrary incident polarization. Mathematically, the generalized radially polarized beam can be expressed as

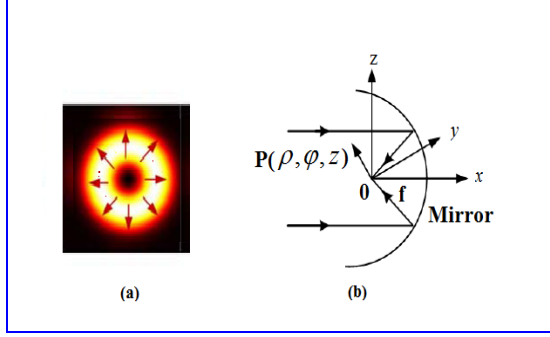
$$E(\rho, \varphi) = l_0(r)(\cos \varphi_0 e_\rho + \sin \varphi_0 e_\varphi) \exp(in\varphi) \rightarrow (1)$$

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where  $l_0(r)$  is complex amplitude distribution of the beam,  $\exp(in\varphi)$  is a helical phase term and  $n$  the topological charge. The Richards–Wolf vectorial diffraction theory, the electric field near the focus at point P can be obtained



**Fig. 1: (a) Schematic of radially polarized annular multi Gaussian beam. (b) Diagram of the focusing configuration.**

$$E^s = -\frac{ikf}{2\pi} \int_{\alpha_0}^{\alpha} \int_0^{2\pi} E_f(\theta, \varphi) e^{ik_s \cdot r_p} \sin \theta d\theta d\varphi \rightarrow (2)$$

Where  $E_f$  is the electric field at the focal sphere, which can be calculated by geometrical optics rules (Zhan *et al.* 2002)

$$E_f = -\frac{2l_0(\theta)}{1+\cos\theta} \cos\theta \cos\varphi e_x + \cos\theta \sin\varphi e_y + \sin\theta e_z \rightarrow (3)$$

where  $2/(1+\cos\theta)$  and  $l_0(\theta)$  are the apodization factor and pupil apodization function for the parabolic mirror, respectively

$$e^{ik_s \cdot r_p} = \exp[-(ikz_s \cos\theta) - ik\rho_s \sin\theta \cos(\varphi - \varphi_s)] \rightarrow (4)$$

where  $k$  is wave number and  $f$  the focal length. Substituting Eq. (3) and Eq. (4) into Eq. (2) and transforming cartesian coordinates to cylindrical coordinates, the local radial, and longitudinal components of  $E_s$  can be calculated as follows

$$E_\rho^s(\rho_s, \varphi_s, z_s) = \frac{(-i)^n kf}{2} \exp(in\varphi) \int_{\alpha_0}^{\theta_{\max}} \frac{\sin 2\theta l_0(\theta)}{1+\cos\theta} A(\theta) [J_{n+1}(k\rho \sin\theta) - J_{n-1}(k\rho \sin\theta)] \exp(-ikz_s \cos\theta) d\theta \rightarrow (5)$$

$$E_z^s(\rho_s, \varphi_s, z_s) = (-i)^{n+1} kf \exp(in\varphi) \int_{\alpha_0}^{\theta_{\max}} \frac{2\sin^2\theta l_0(\theta)}{1+\cos\theta} A(\theta) J_n(k\rho \sin\theta) \times \exp(-ikz_s \cos\theta) d\theta d\varphi \rightarrow (6)$$

$A(\theta)$  describes the amplitude-modulated annular multi-Gaussian beam, this function is given by (Nong *et al.* 2011)

$$A(\theta) = \left(\frac{\theta}{\theta_0}\right) \sum_{n=-N}^N \exp\left[-\left(\frac{\theta - \theta_c - n\omega_0}{\omega_0}\right)^2\right] \rightarrow (7)$$

Here,  $\theta$  is the converging semi-angle. We denote the maximum converging semi-angle as  $\theta_{\max}$  which is related to objective numerical aperture by  $\theta_{\max} = \arcsin(\text{NA})$ . Here we take  $\theta_c = \theta_{\max}/2$ .  $w_0$  is the waist

width of single Gaussian beam which is calculated by the following formula

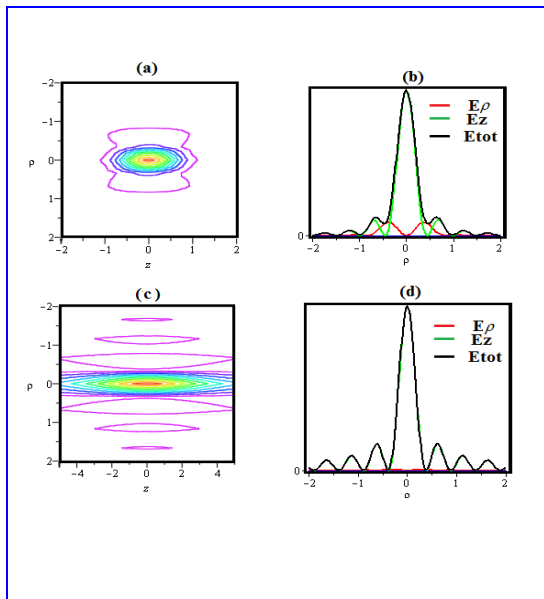
$$\omega_0 = 1/2 \times \frac{\theta_{\max}}{N + \left\{1 - \ln\left[\sum_{n=-N}^N \exp(-n^2)\right]\right\}^{1/2}} \rightarrow (8)$$

### 3. RESULTS

We perform the integration of Eq.(1) numerically for  $\text{NA}=0.9$  and  $\lambda=1$ ,  $\theta_{\max}=64^\circ$ ,  $\theta_0=62^\circ$  and  $\theta_c=\theta_{\max}/2$ ,  $w_0=0.0364$ ,  $N=20$  and  $m=20$ . For all calculation in the length unit is normalized to  $\lambda$  and the energy density is normalized to unity. Fig. (2) illustrates the evolution of three-dimensional light intensity distribution of high NA mirror for the incident plane radially polarized beam. From the fig.(2.a), it is observed that for incident plane radially polarized beam, the generated focal segments is a focal spot with focal depth of  $1.2\lambda$ . From the Fig. (2.b), we measure the radial component is 10% of the total intensity of the total intensity and the FWHM of the focal spot is  $0.75\lambda$ . We observed that it is possible to generated sub wave length focal spot segment with large focal depth using annular multi Gaussian beam. It is observed from the Fig.2(d) and 2.(c), that the FWHM of the generated focal spot for the incident annular multi Gaussian beam is about  $0.4\lambda$  and its focal depth improved as  $9.5\lambda$  respectively. The generated focal depth is much larger than the incident plane radially polarized beam. More over it is observed from the Fig. (2.d), that the radial component is only 4% of the total intensity.

Hence we can achieve highly confined longitudinal focal field with very large focal depth which finds application in particle trapping, data storage, biomedical imaging, laser drilling, and machining. Such a needle of sub wavelength beam with

large focal depth finds its application in near field optical recording and particle acceleration.



**Fig. 2: (a) Contour plot for the total intensity distribution in the  $\rho$ - $z$  plane for radially polarized parabolic mirror. (b) 2D Intensity distributions at the focus of the mirror for  $z=0$ . (b) & (d) are same as (c) and (d) but for radially polarized annular multi Gaussian beam**

#### 4. CONCLUSION

In conclusion, based on the vector diffraction theory, the focusing property of high NA parabolic mirror for the incident radially polarized annular multi Gaussian beams is studied numerically using vector diffraction theory. It is observed that highly confined sub wavelength longitudinal focal spot with large focal depth. So the high NA parabolic mirror can be an especially useful alternative for focusing and beam-shaping. Such a sub wavelength focal structures with large focal depth can be widely used in applications such as data storage, biomedical imaging, laser drilling and machining.

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