



## **Creation of needle of transversely polarized beam in vacuum by using complex phase plate**

**K. Lalithambigai<sup>1</sup>, C. Mohana Sundaram<sup>1</sup>, P. M.Anbarasan<sup>1</sup>, K. B.Rajesh<sup>2,\*</sup>**

<sup>1</sup>*Department of Physics, Periyar University, Salem, TN, India.*

<sup>2</sup>*Department of Physics, Chikkanna Government Arts College, Trippur, TN, India.*

*Received:02.06.2014 Accepted:22.08.2014*

### **Abstract**

*The intensity distribution in the focal region for the tight focusing of double-ring-shaped azimuthally polarized beam with complex phase plate is studied on the basis of the vector diffraction theory. Here we report a new method that generates a transversely polarized light beam with sub diffraction beam size ( $0.488 \lambda$ ) that propagates without divergence over a long distance (of about  $8.6 \lambda$ ) in free space. This work is important for optical manipulation and may find applications when using optical materials or instruments responsive to the transversal field only.*

**Keywords:** *Vector diffraction theory; High NA lens; Complex phase plate; Transversely polarized beam.*

### **1. INTRODUCTION**

The radial polarization beam was once to be regarded as a more effective way to achieve a smaller focal spot than other polarized beams focused by high NA aplanatic lens due to the dominant longitudinal component (K. S. Youngworth et al. 2000). However, recently it is proved that the azimuthally polarized beam phase encoded by a vortex  $0-2\pi$  phase would produce a significantly sharper spot on the focal plane (X. Hao et al. 2010). It is also proved that an annular azimuthally polarized beam with proper phase modulation can still achieve a further reduction (about 13.5%) to the spot area, compared with the radial polarization considered in previous papers. Recently, a non diffracting “dark channel” with a long DOF was recently achieved by tight focusing of a double-ring-shaped azimuthally polarized beam (B. Tian et al. 2011; K. Lalithambigai et al. 2012). More recently the generation of a non diffracting transversally polarized beam by highly focusing an azimuthally polarized beam with a high-NA lens and a multibelt spiral phase hologram is demonstrated numerically (G. H. Yuan et al. 2011). It is reported that the beam propagates without divergence and its polarization keeps almost invariant in the main lobe along the optical axis over a relatively long distance ( $\sim 4.84 \lambda$ ). Such a beam may

find applications when using optical materials or instruments responsive to the transversal field only. The objective of the study reported in this paper is to investigate the effect of Complex Phase Plate [CPP] in the focal region of an azimuthally polarized Laguerre-Gaussian (1, 1) beam theoretically. We observe that our proposed system can generate a transversely polarized subwavelength ( $0.488\lambda$ ) focal spot with large focal depth ( $8.6\lambda$ ).

### **2. PROPOSED SET UP AND MODELING**

A schematic configuration of the suggested method is shown in Fig 1. The analysis was performed on the basis of Richards and Wolf’s vectorial diffraction method widely used for high NA focusing systems at arbitrary incident polarization (B. Richards et al. 1959). An incident azimuthally polarized Laguerre-Gaussian beam transmits through a CPP and is subsequently focused by a high NA lens. Shown in Fig. 1, if a complex pupil plate is placed at the pupil plane, where the polarized states of the incident beams in the area of inner and outer rings are reverse, so as to cause the destructive interference in the focal region. In the case of the azimuthally incident polarization, adopting the cylindrical coordinates  $r, z, \varphi$  and the notations [1], the electric field  $E(r, \varphi, z)$  in the vicinity of the focal region can be written as,

\* **K.B.Rajesh** Tel.: +919942460031  
E-mail: [rajeskb@gmail.com](mailto:rajeskb@gmail.com)

$$E(r, \varphi, z) = \begin{bmatrix} E_r \\ E_\varphi \\ E_z \end{bmatrix} = \begin{bmatrix} -Ae^{i\phi} (I_0 + I_2) \\ -iAe^{i\phi} (I_0 - I_2) \\ 0 \end{bmatrix} \quad (1)$$

where

$$I_n = \int_0^{\theta_m} \cos^2(\theta) \sin(\theta) F(\theta) l_0(\theta) e^{ik_0 z \cos \theta} J_n(k_0 r \sin \theta) d\theta \quad (2)$$

where  $A$  is relative amplitude,  $\theta_m = \arcsin(NA/n)$  that is the maximum aperture angle with  $(NA/n)$  is the numerical aperture and  $n$  is the index of refraction between the lens and the sample.  $k_0$  is the wave number in free space.

$J_n(\theta)$  denotes the  $n$ th-order Bessel function of the first kind, and the function of  $l_0(\theta)$  describes the amplitude modulation. For illumination by a double-ring-shaped R-TEM<sub>11</sub>\* beam with its waist in the

pupil, this function is given by [A. A. Tovar et al. 1998; Y. Kozawa et al. 2006]

$$l_0(\theta) = \beta^2 \frac{\sin \theta}{\sin^2 \theta_m} \exp \left[ - \left( \beta \frac{\sin \theta}{\sin \theta_m} \right)^2 \right] L_p^1 \left[ 2 \left( \beta \frac{\sin \theta}{\sin \theta_m} \right)^2 \right] \quad (3)$$

where  $\beta$  is determined by the ratio of the pupil radius to the waist of the incidence beam and  $L_p^1$  is the generalized Laguerre polynomial. If  $p = 1$ , the incident azimuthally polarized beam is a double-ring-shaped azimuthally polarized beam. It is seen from Eq. (1) that an additional radial electric field component is produced after introducing the phase plate (G. H. Yuan et al. 2011). Owing to this radial component, the polarization near the focal plane is rather complicated and space variant.

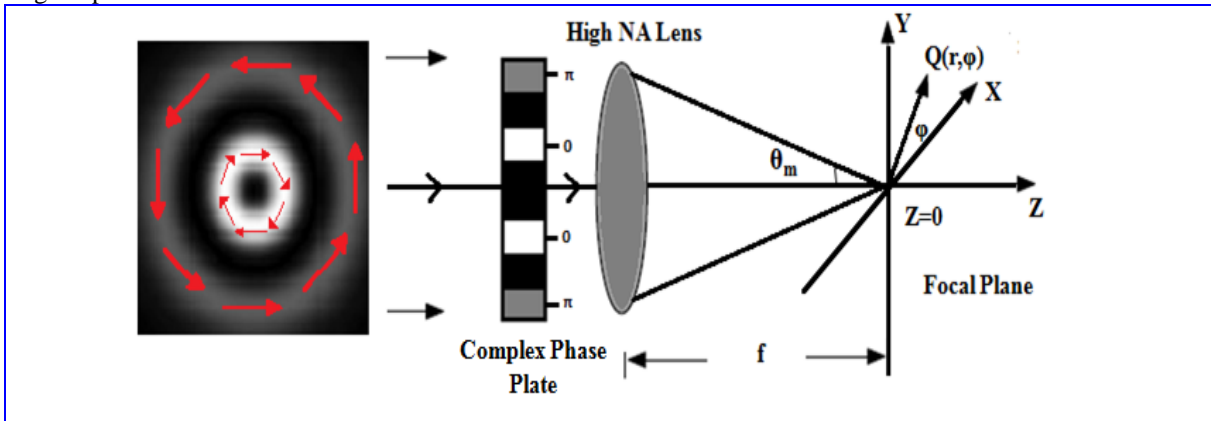


Fig. 1: Schematic Diagram of the proposed system. Azimuthally polarized double-ring-shaped beam passes through a complex Phase Plate and is subsequently focused by a high NA lens.

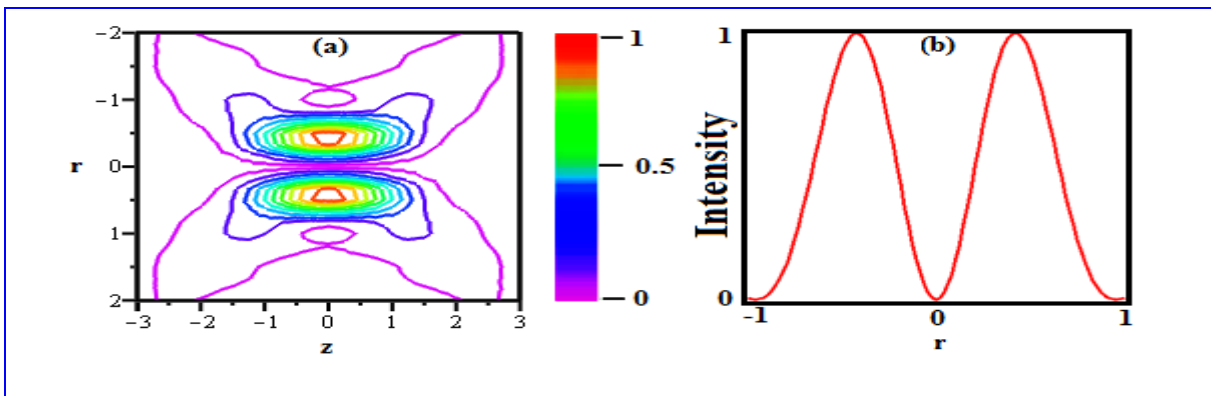


Fig. 2: (a) Shows the contour plot of the intensity profile of the azimuthally polarized beam focused by high NA lens. (b) The results of electric field component intensity distributions in the radial direction.

### 3. RESULTS AND DISCUSSION

We perform the integration of Eq. (2) numerically using parameters  $\lambda=1$ ,  $\beta=1.01$  and NA of the objective is 0.95. Here, for simplicity, we assume that the refractive index  $n = 1$  and  $A = 1$ . The CPP consists the combination of amplitude and phase plate and it has 4 belts in the radial direction. A CPP is placed at the pupil plane, where the transmissions from the inner to the outer zones are 0, 1, 0, and -1 respectively. The effect due to the interference of the second and fourth zones of the CPP on the input double ring shaped azimuthally polarized beam is evaluated the function  $F(\theta)$ , where  $F(\theta)$  is given by [H. Guo et al. 2011, J. Cao et al. 2011]

$$f(\theta) = \begin{cases} 1, & \text{for } 0 < \theta < \theta_1, \theta_2 < \theta < \theta_3, \\ 1, & \text{for } \theta_1 < \theta < \theta_2, \\ -1, & \text{for } \theta_3 < \theta < \theta_m \end{cases} \quad (4)$$

When a double-ring shaped azimuthally polarized beam without any phase modulation is tightly focused, a single ring focal hole pattern is obtained. The calculated electric field distribution in the focal plane in the case of without phase plate is shown in Fig. 2. The FWHMs for  $|E_\phi|^2$  in the radial direction of tight focusing of azimuthally polarized beam is  $0.430 \lambda$ , respectively, while the DOF is approximately  $2.1 \lambda$  as shown in Figs.2(b) and 2(a). Based on the above explicit formula, we aim to achieve a long DOF of transversally polarized beam by utilizing the CPP function  $F(\theta)$ .

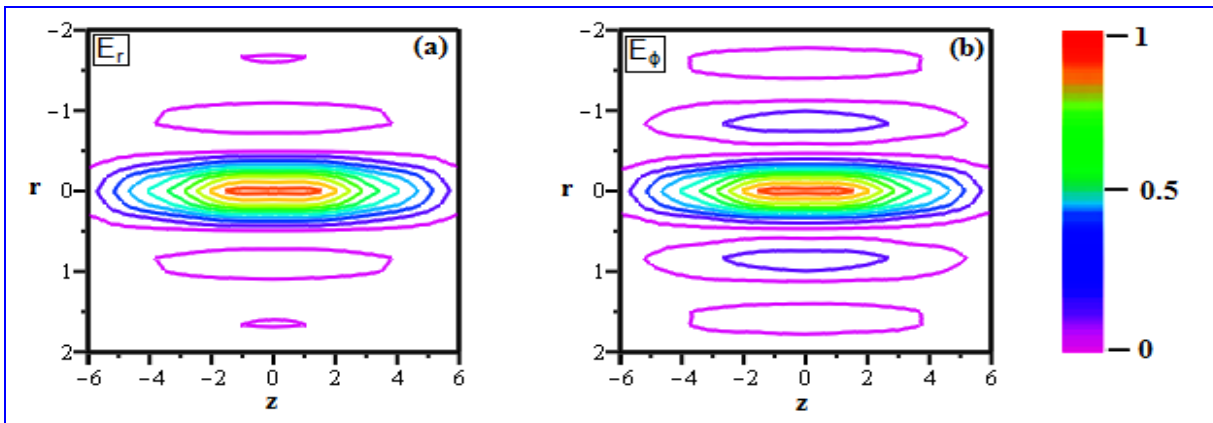


Fig. 3: Shows the intensity profile of the (a) radial and (b) azimuthal components of the optical field at focus of high NA lens with dedicated CPP.

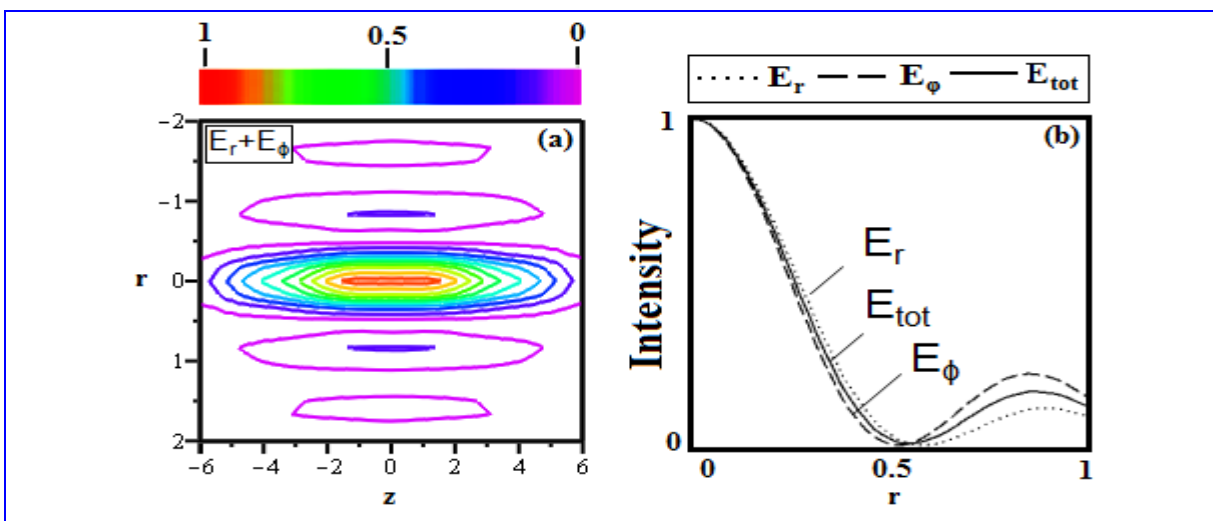
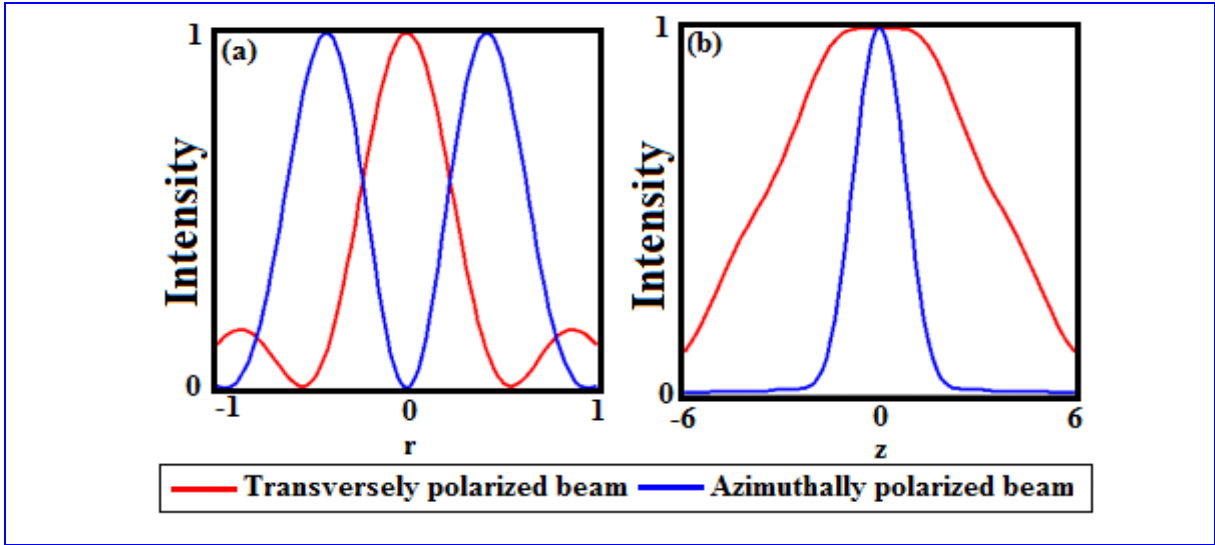


Fig. 4: Shows the intensity profile of the (a) total electric field component of the optical field at focus of high NA lens with dedicated CPP. (b) The results of electric field component intensity distributions in the radial direction.



**Fig. 5: (a) FWHM of the focal hole of the azimuthally polarized beam and the focal spot of the transversely polarized beam with dedicated CPP (b) shows the DOF of the focal hole of the azimuthally polarized beam at  $r=0.45 \lambda$  and the focal spot of the transversely polarized beam at  $r=0$  with dedicated CPP.**

A transversally polarized beam with a relatively long DOF and a bright beam spot can be generated by highly focusing an azimuthally polarized beam after CPP is shown in Fig. 3 and Fig. 4. The corresponding radial and azimuthal components of the electric field intensity distribution of the focal spot are shown in Figs. 3(a) and 3(b). The FWHMs for  $|E_r|^2$  and  $|E_\phi|^2$  in the transversal cross section are  $0.57 \lambda$  and  $0.51 \lambda$ , respectively. Fig. 4 shows the total intensity distribution of the transversally polarized beam spot segment at the axial cross section, generated by the proposed high NA lens with combination of dedicated CPP. It is observed from the Fig. 4(a) that the generated transversally polarized focal spot extends up to  $8.6 \lambda$  and the FWHM of the generated focal spot is measured as  $0.488 \lambda$ . The results of electric field component intensity distributions in the radial direction as shown in Fig. 4(b). It is interesting that both  $|E_r|^2$  and  $|E_\phi|^2$  possess bright spots with almost equal intensity at the beam center even though they carry a CPP term  $F(\theta)$ , which is different from an ordinary azimuthally polarized beam, which always features a hollow intensity profile due to the existence of polarization singularity. The set of three angles of the CPP optimized to achieve the above mentioned focal spot segment are  $\theta_1 = 39.62^\circ$ ,  $\theta_2 = 49.49^\circ$  and

$\theta_3 = 66.90^\circ$ . These angles are optimized by choosing one structure with random values for  $\theta_1$  to  $\theta_3$  from all possibilities and simulate their focusing properties by vector diffraction theory. If the structure generates a sub wavelength focal spot structure and satisfies the limiting condition, that the FWHM of the generated focal spot size is less than  $0.65 \lambda$ , it is chosen as the initial structure during the optimization procedures. In the following steps, we continue to vary  $\theta$  of one chosen zone to improve the focal depth without changing the limiting condition. The value of the newly chosen zone thickness is used in the next step. Then, we randomly choose the other zones and repeat these procedures to improve the focal depth and uniformity of the generated focal spot segment.

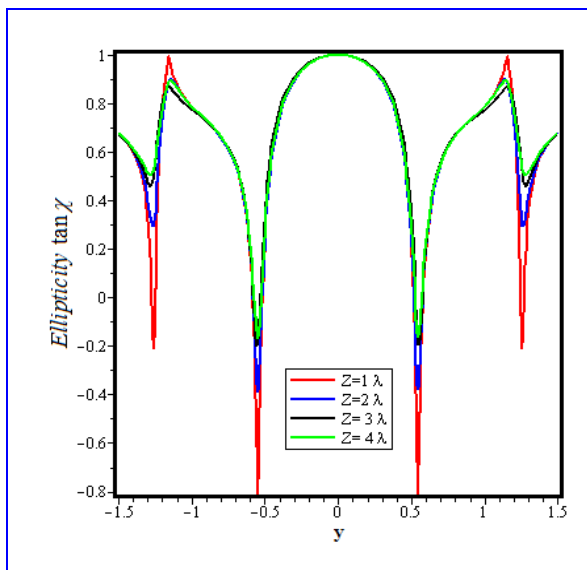
It is observed from the normalized axial intensity, the FWHM of the transversally polarized focal spot generated by the proposed CPP is almost 1.13 times greater than the focal hole generated by an azimuthally polarized beam (without phase plate) as shown in Fig. 5(a). The DOF inferred from the normalized axial electric field intensity, as shown in Fig. 5(b) (solid red line), is approximately  $8.6 \lambda$ , which is 4.09 times longer than that achieved by using an azimuthally polarized beam without phase plate (solid blue line). We also studied the polarization evolution of this nondiffracting beam by calculating the Stokes polarization parameters [Z. Bomzon et al.

2006] from which the azimuthal angle  $\psi$  and ellipticity  $\tan \chi$  of the polarization ellipse can be derived:

$$\tan 2\psi = S_2/S_1 = \tan[2\phi + \arg(I_0^* I_2)] \quad (5)$$

$$\sin 2\chi = S_3/S_0 = (|I_0|^2 - |I_2|^2) / (|I_0|^2 + |I_2|^2) \quad (6)$$

where  $S_i$  describes the Stokes polarization vector component, the superscript symbol  $*$  denotes the complex conjugate, and  $\arg(x)$  gives the argument of the complex number  $x$ . The ellipticities of the local polarization ellipses at different  $z$  are shown in Fig. 6. The polarization has no apparent changes within the DOF range, especially in the mainlobe where most of the optical energy is concentrated when using CPP. This reveals that not only the electric field intensity but also the polarization of the nondiffracting beam keeps almost invariant within the DOF range. Thus, it is possible to generate subwavelength transversally polarized focal spot segment for the input double-ring-shaped azimuthally polarized beam.



**Fig. 6: Cross section of the ellipticity of local polarization ellipses at different observation planes for transversely polarized beam.**

#### 4. CONCLUSION

In conclusion, the proposed CPP with high NA lens to generate a transversally polarized beam by focusing a double-ring-shaped azimuthally polarized beam is analyzed and demonstrated

numerically by using vector diffraction theory. The proposed method generated a sub wavelength transversely polarized focal spot having FWHM of  $0.488 \lambda$  with long focal depth of  $8.6 \lambda$ . The authors expect such a needle of transversely polarized beam may find its application when using optical materials or instruments responsive to the transversal field only.

#### ACKNOWLEDGMENTS

One of the authors, K. Lalithambigai expresses her sincere thanks to UGC-BSR, New Delhi, India (UGC Letter No. 11-142/2008(BSR)) for providing financial support.

#### REFERENCES

- Bomzon, Z., Gu, M., Shamir, Angular momentum and geometrical phases in tight-focused circularly polarized plane waves, *Appl. Phys. Lett.* 89, 2411-2421 (2006).  
doi:10.1063/1.2402909
- Cao, J., Chen, Q. and Guo, H., Creation of a controllable three dimensional optical chain by TEM01 mode radially polarized Laguerre–Gaussian beam, *Optik* 124, 2033-2036 (2013).  
Doi:10.1016/j.ijleo.2012.06.057
- Guo, H., Weng, X., Dong, X., Sui, G., Gao, X. and Zhuang, S., Three dimensional optical cage formed by TEM01 mode radially polarized Laguerre-Gaussian beam, *J. Opt.*, 40, 206–212 (2011).  
doi:10.1007/s12596-011-0055-8
- Hao, X., Kuang, C., Wang, T. and Liu, X., Phase encoding for sharper focus of the azimuthally polarized beam, *Opt. Lett.* 35, 3928-3930 (2010).  
doi:10.1364/OL.35.003928
- Kozawa, Y. and Sato, S., Focusing property of a double-ring-shaped radially polarized beam, *Opt. Lett.* 31, 820-822 (2006).  
doi:10.1364/OL.31.000820
- Lalithambigai, K., Suresh, P., 173 Ravi, V., Prabakaran, K., Jaroszewicz, Z., Rajesh, K. B., Anbarasan, P. M., Pillai, T. V. S., Generation of sub wavelength super-long dark channel using high NA lens axicon, *Opt. Lett.* 37, 999–1001 (2012).  
doi:10.1364/OL.37.000999
- Richards, B., Wolf, E., Electromagnetic diffraction in optical systems, II. Structure of the image field in an aplanatic system, *Proc. R. Soc. Lond. A Math. Phys. Sci.* 253, 358– 379 (1959).  
doi:10.1098/rspa.1959.0200

- Tian, B. and Pu, J., Tight focusing of a double-ring-shaped, azimuthally polarized beam, *Opt. Lett.* 36, 2014-2016 (2011).  
[doi:10.1364/OL.36.002014](https://doi.org/10.1364/OL.36.002014)
- Tovar, A. A., Production and propagation of cylindrically polarized Laguerre–Gaussian laser beams, *J. Opt. Soc. Am. A* 15, 2705-2711 (1998).  
[doi:10.1364/JOSAA.15.002705](https://doi.org/10.1364/JOSAA.15.002705)
- Youngworth, K. S. and Brown, T. G., Focusing of high numerical aperture cylindrical vector beams, *Opt. Express* 7, 77–87 (2000).  
[doi:10.1364/OE.7.000077](https://doi.org/10.1364/OE.7.000077)
- Yuan, G. H., Wei, S. B., Yuan, X. C., Nondiffracting transversally polarized beam, *Opt. Lett.* 36, 3479-3481 (2011).  
[doi:10.1364/OL.36.003479](https://doi.org/10.1364/OL.36.003479)