Research Article

Journal of
Optics and Photonics
Technology

Application of Vector Theory to Study Azimuthally Polarized Phase Modulated DG Beam through a Dielectric Interface

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Received: 12.11.2024 Accepted: 08.12.2024 Published: 30.03.2025
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ABSTRACT

Vector diffraction theory is used to theoretically examine the tight focusing characteristics of an azimuthally polarized doughnut Gaussian beam focusing *via* a dielectric contact. According to numerical calculations, an incident doughnut Gaussian beam that is azimuthally polarized can have a sub-wavelength focus pattern. As a result, in the super-resolution focusing region, the azimuthally polarized doughnut Gaussian beam is crucial. Additionally, we noticed that a subwavelength focal hole segment with a significant focal depth is formed for the annular obstruction with δ=0.5 and δ=0.75. We have demonstrated that, with a well-designed multi belt complicated phase filter, it is possible to generate numerous subwavelength focal hole segments in addition to a single focal hole with a lengthy depth of focus. It is shown that 3D optical chains with 10 focal hole segments in the focal region may be produced using the MBCPF's appropriately tuned parameters.

Keywords: Focusing properties; Azimuthally polarized doughnut Gaussian beam; Dielectric interface.

1. INTRODUCTION

Numerous applications, including microscopy (Chon *et al.* 2005; Ganic *et al.* 2003; Mumro *et al.* 2010), optical tweezing (Ganic *et al.* 2005), Raman spectroscopy (Hayazawa *et al.* 2004), fluorescent imaging (Novotny *et al.* 2003), and particle acceleration (Romea *et al.* 1990), make the tight focusing effect of light an important area of study. This effect is made use in many real-world applications involving with varying refractive indices. For example, in semiconductor inspection, light beams are focused from air onto silicon substrates, and in laser trapping, a light beam is focused across a glass-water interface. A theoretical approach for examining the focussing of an electromagnetic wave over dielectric surfaces was established by Torok *et al.* (1995a; 1995b; 1995c). In 1995a, 1995b, and 1995c, Torok *et al.* Biss and Brown talked about employing a dielectric interface to focus cylindrical vector beams (CVBs) (Van *et al.* 2011). Tight focusing characteristics of CPL (circularly polarized luminescence) in insulating media have been investigated (Mansuripur *et al.* 2005).

Zhang *et al.* (2008a; 2008b) demonstrated the tight focusing characteristics of vortex beams that are polarized in both directions through a uniaxial birefringent crystal and a dielectric interface, respectively. Doughnut Gaussian (DG) beams are a novel type of beam that was recently introduced in a high NA focusing system. Similar works with radial polarization are available in the literature (Kitamura *et al.* 2010; Liu *et al.* 2007; Prabakaran *et al.* 2014: Udhayakumar *et al.* 2018). It has been shown earlier that doughnut Gaussian beams can be tightly focused to create longitudinal magnetization patches and transversely polarized focal fields (Gu 2000; Born 1999). and. In this study, using the approach (Gu, 2000) based on (Born, 1999), we numerically illustrate the focusing performance of the azimuthally polarized DG beam focused across a dielectric interface.

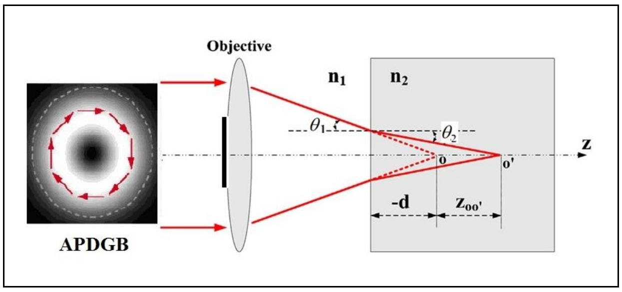

Fig. 1: Scheme of focusing optical system

2. THEORY

Fig. 1 shows a schematic representation of the suggested optical system. The DG beam's electric field at the output pupil for a high NA lens is defined as

$$A(\Theta) = \exp\left[-\left(\frac{\sin(\theta)-\theta_0}{w_0}\right)\right] \quad \dots (1)$$

The function's variable is denoted by θ. θ_0 is related to the radius of the DG beam. w_0 determines the width while θ_0 determines the location of the highest field strength. Thus, these two parameters define the form of the DG beam.

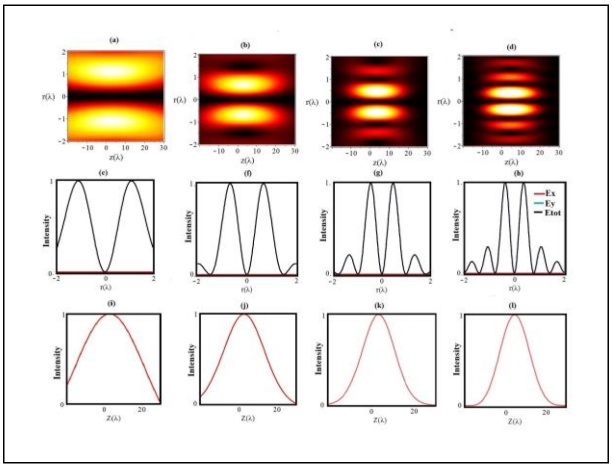

Fig. 2: Intensity distribution curves

The electric field vector in the focal region of incident azimuthally polarized beams with Cartesian components is determined by (Youngworth *et al.* 2000)

$$E(r,\psi,z)=\begin{bmatrix}E_x(r,\psi,z)\\E_y(r,\psi,z)\\E_z(r,\psi,z)\end{bmatrix}$$
$$=\frac{-iE_0}{\pi}\int_{\delta\cdot\alpha}^{\alpha}\int_{0}^{2\pi}\exp[-ik_0\Phi(\theta_1,\theta_2)]\times\sin\theta_1\sqrt{\cos\theta_1}E_0(\theta)\times$$
$$t_s\exp[ik_2z\cos\theta_2+ik_1r\sin\theta_1\cos(\psi-\phi)]\times\begin{bmatrix}-\sin\phi\\\cos\phi\\0\end{bmatrix}d\phi d\theta_1\rightarrow(2)$$

where the symbols denote the usual parameters. t_p and t_s are the amplitude transmission coefficients for parallel and perpendicular polarization states, which is given by the Fresnel equations (Born, 2000)

$$t_s=\frac{2\sin\theta_2\cos\theta_1}{\sin(\theta_1+\theta_2)} \quad \dots (3)$$

The function $\Phi(\theta_1, \theta_2)$ is given by $\Phi(\theta_1,\theta_2)=-d(n_1\cos\theta_1-n_2\cos\theta_2$. The well-known Snell rule relates θ_1 and θ_2.

3. RESULTS AND DISCUSSION

It is suggested that the numerical aperture of the focusing optical system NA = 0.95 and relative waist width be used in the study of azimuthally polarized doughnut Gaussian beam focusing over a dielectric interface without sacrificing validity and generality. Here, we suppose that λ = 632.8 nm, n_1 = 1, and n_2 = 3.55 mm, d = 1 λ are the other parameters.

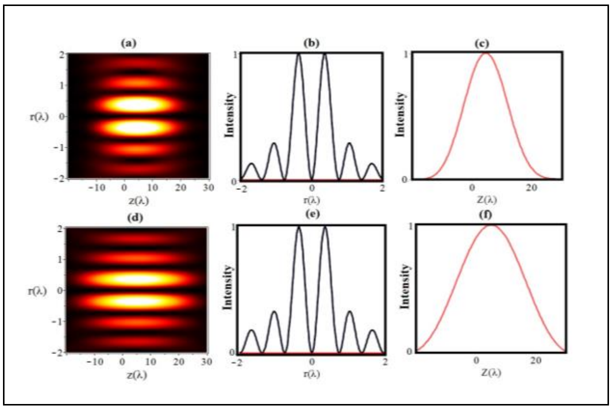

Fig. 3: Intensity distributions in the focal plane as a doughnut Gaussian beam through a dielectric interface using annular obstruction δ=0.5 and 0.75

The energy density was standardized to unity and the wavelength was normalized to λ for all calculations. The self explanatory figure (Fig. 2) shows the various intensity distribution curves of the DG beam. The focal hole segment's FWHM and focal depth are observed to decrease when the θ_0 is increased from 0.2 to 0.8. Table 1 displays the focal depth and FWHM for the various values of θ_0. It is observed that the resulting focal hole's DOF is 44.2 λ and its FWHM is 1.22 λ when θ_0=0.2. However, the FWHM is significantly reduced to 0.362 λ and the focal depth is lowered to 14.9 λ when θ_0 is raised to 0.8.

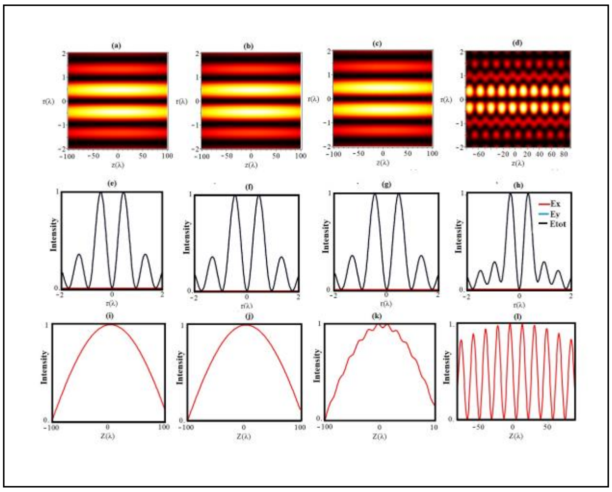

Fig. 4: 2D-Normalized intensity distribution curves

Table 1. Focal depth and FWHM for the various values of θ_0

θ_0	FWHM (λ)	DOF (λ)	Position of maximum Intensity
0.2	0.360	113.8	3.80
0.4	0.360	104.0	3.80
0.6	0.350	94.70	10.4
0.8	0.345	9.400	12.4

Therefore, we introduce annular obstruction for the DG beam with θ_0 for 0.8 in order to further improve the focal depth without lowering the FWHM of the created focal hole. The 3D intensity in the r-z plane for the annular obstacle with δ=0.5 and the DG beam with $\theta_0 = 0.8$ is displayed in Fig. 3. The existence of annular obstruction is further shifted to 5.1 λ for the position of greatest intensity. Additionally, the created focal hole's focal depth improved to 15.7 λ while its FWHM decreased to 0.308 λ. Raising the annular obstacle δ to 0.75 raises the maximum intensity position to 6.3λ.

It is observed that the focal depth has increased to 25.5 λ and the FWHM of the resultant focal hole segment has further decreased to 0.30 λ. Therefore, the focal depth of the created focal structure can be further enhanced by employing annular blockage. In STED microscopy, a needle with a subwavelength focal hole and an extended focal depth is helpful for erasing beams and capturing particles with low refractive indices (Youngworth *et al.* 2000; Bingen *et al.* 2011). Here, we examine the use of a complex phase filter (CPF) in an effort to further enhance the focal depth and the potential for producing several focal hole segments. Azimuthally polarized doughnut input and the impact of a multi-belt complicated phase filter The Gaussian beam is assessed by substituting P(θ)X MBCPF (θ) for the function P(θ) as provided by (Wang *et al.* 2012),

$$MBCPF(\theta) = \begin{cases} 0, \ for 0 < \theta < \theta_1, \theta_2 < \theta < \theta_3, \\ 1, \ for \theta_1 < \theta < \theta_2, \\ -1 \ for \theta_3 < \theta < \theta \max \end{cases} \quad \ldots (4)$$

Here, the four-belt spiral phase hologram is taken into consideration, and the typical global-search-optimization technique is used to optimize the set of four angles in order to produce a specific focal pattern.

For example, $\theta_1 = 41.27°$, $\theta_2 = 42.42°$, $\theta_3 = 56.17°$, and $\alpha = 71.84°$ are the four angles that are optimum for homogenous sub wavelength focal hole segments. Fig. 4 displays the 2D-normalized intensity distribution curves. We observed the optimal CPF-generated focal segment with a focal depth of 104 λ and 94.7 λ and an FWHM of 0.36λ and 0.35, which correspond to $\theta_0 = 0.4$ and 0.6. However, we observed that increasing $\theta_0 = 0.8$ further produced a focal segment with 10 focused holes. There is an axial separation of 12.4 λ between each focal hole. For both STED microscopy with screening rate and repeated trapping of low refractive index particles, such a chain of focus hole segments is helpful (Youngworth *et al.* 2000). Therefore, we show here that a single high NA focusing unit with a well-optimized CPF may be used to create a chain of hole holes for varying values of θ_0 of a doughnut Gaussian beam, as well as a subwavelength scale focal hole segment with configurable focal depth.

4. CONCLUSION

We used vector diffraction theory to analyse the properties of DG beams. According to numerical studies, an azimuthally polarized doughnut Gaussian beam can achieve sub-wavelength focusing. We have demonstrated the ability to adjust the focal depth and produce many subwavelength focal hole segments using a well-designed multi belt complicated phase filter, in addition to producing a single focal hole with a lengthy depth of focus. We anticipate that this theoretical prediction will have useful applications in optical engineering, particle trapping, microscopy, and other fields.

FUNDING

There is no funding source.

CONFLICT OF INTEREST

The authors declared no conflict of interest in this manuscript regarding publication.

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